

# The Ontology of Residue

A Philosophy of Structural Non-Describability and  
a Theory of Minimal Descriptive Extension

残余の存在論

——虚次元の不可記述性の哲学、および記述の最小拡張の理論——

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i . F r a g m e n t a

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## Abstract

Why does extension in mathematics terminate, while extension in ontological description does not? The field of real numbers, confronted with its failure to be algebraically closed, was closed by the addition of a single axis: the imaginary unit  $i$ . The description of the real dimension, by contrast, does not close even once the imaginary dimension  $iD$  has been acknowledged. This paper locates the ground of that difference in the recursivity of extension — the structure whereby a description that acknowledges a non-describable position generates, with each acknowledgement, a further residue.

As the sixth paper of the Extended Imaginary Number Theory series, it undertakes two tasks. First, it shows that the self-limitation running through the preceding five papers — the mode of description that never describes the imaginary dimension directly but requires its existence structurally — accumulated across five stages, and that this accumulation functioned not as a negative brake but as the condition enabling entry into deeper territory. Second, it formalises the operation at the core of that mode as a theory of residue. Minimal extension, established as a principle in Paper V — a description that fails to close requires an independent axis, and only the minimum required — divides in two by the criterion of whether the introduced axis is itself describable: into instrumental extension and residual acknowledgement. Only the former can bring a system to closure (Proposition P1a); the latter necessarily recurs (Proposition P1b). The three operations the series has performed — the addition of the imaginary unit  $i$ , the requirement of imaginary time  $it$ , and the acknowledgement of the imaginary dimension  $iD$  — are unified under this single classification, and structural simultaneity, left by Paper V as a task for subsequent work, is likewise given as a corollary of the theory; its application is restricted to systems whose antecedent has been argued for (the real time axis  $t$ ).

The conclusion of this paper contracts into a single proposition: a descriptive system can be extended beyond itself only in virtue of failing to close, and whether that extension terminates or recurs is settled by whether the required residue itself becomes describable.

**Keywords:** non-describability, structural surplus, principle of minimal extension, instrumental extension, residual acknowledgement, recursivity, structural simultaneity, imaginary dimension, structural avoidance, extended imaginary number theory

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## 01 Introduction — The Governing Proposition and the Five-Paper Configuration

### 1.1 The Governing Proposition and the Place of This Paper in the Series

The claim running through this paper contracts into a single proposition.

#### [Governing Proposition]

A descriptive system can be extended beyond itself only in virtue of failing to close. And whether that extension terminates or recurs is settled by whether the required residue itself becomes describable.

The first half — that failure to close is the sole ground of extension — summarises the principle of minimal extension established in Paper V, *Imaginary Time* (Muranushi, 2026e, Principle 1). The second half — that whether the residue becomes describable settles the fate of the extension — is this paper’s own contribution (Proposition P1, §05).

The configuration of the five papers preceding this one is given in Table 1.

[Table 1 Theoretical map of the Extended Imaginary Number Theory series]

| Paper                                   | Domain             | Core formulation / concept                           | Methodological position (this paper’s vocabulary, §04–05)   |
|-----------------------------------------|--------------------|------------------------------------------------------|-------------------------------------------------------------|
| I Extended Imaginary Number Theory      | Ontology           | $Z = D + iD$                                         | Prototype of residual acknowledgement                       |
| II The Structural Theory of Occurrence  | Phenomenology      | Occurrence ( <i>okori</i> )                          | Real-dimension-side description of the acknowledgement      |
| III Tōh (The Prayer of Becoming)        | Existential theory | $\Phi = P \times K \cdot V(Z) = \alpha D + \beta iD$ | Acknowledgement within a formal apparatus                   |
| IV The Far North of Real-Dimensionalism | Response           | Descriptive non-closure                              | Controlled suspension                                       |
| V Imaginary Time                        | Theory of time     | $T = t + it \cdot$ principle of minimal extension    | Instrumental extension; explicit statement of the principle |

Only the founding formulation need be restated here: Paper I describes the totality of an object’s being,  $Z$ , as the superposition of a real-dimension component  $D$ , which appears in description through observation, verbalisation and conceptualisation, and an imaginary-dimension component  $iD$ , the structural surplus that description never exhausts —  $Z = D + iD$  (Muranushi, 2026a). The mode of description shared by Papers I to III — never describing the imaginary dimension directly, but requiring its existence structurally — was suspended under control by Paper IV, which argued for non-closure (§4.2); Paper V then established that mode of derivation as the principle of minimal extension and, in

its conclusion, left the generalisation of that principle and a general theory of structural simultaneity as tasks for subsequent work.

This paper is where those two tasks are taken up. Two things are done here. First, the thematisation of the descriptive mode running through the preceding papers is updated under the five-paper configuration (§02–04). Second, the operation at the core of that mode is formalised as a theory of residue (§05), and structural simultaneity is derived as its corollary (§06). By this means the three operations the series has performed separately — the addition of the imaginary unit  $i$ , the requirement of imaginary time  $it$ , and the acknowledgement of the imaginary dimension  $iD$  — are unified as three instances of a single theory. From §07 onward, the framework so established is tested by locating it against the family of surplus concepts in the history of philosophy.

## 1.2 Scope and Terminology

This paper does not describe the imaginary dimension directly. What it undertakes is a meta-description, from the side of the real dimension, of the non-describability of the imaginary dimension as such. The use of “ontology” in the title calls for a word of clarification. This paper uses “ontology” not as a metaphysical doctrine asserting what exists, but as a methodological extension of a theory of structural description. “The ontology of residue” is not a doctrine asserting that residue exists; it is a doctrine describing, from within the present framework, what the structural position of residue is. In this respect it belongs to a different level from the Extended Realism advanced in Paper IV — a realist claim that recovers, on the side of the real, the surplus that description leaves behind. Extended Realism is a claim; the ontology of residue is the grammar in which such a claim is written. The two do not conflict.

Terminological notes are kept to a minimum. Throughout this paper “residue” is used in the sense of surplus as structural position (*structural residue*), not surplus as excess (*residual surplus*) — see §8.7. “Extension” denotes the operation of adding an independent descriptive axis to a descriptive system; Definition 2 (descriptive closure), Norm 1 (grounds for positing an axis) and Principle 1 (the principle of minimal extension) of Paper V are inherited exactly as they stand. This paper does not redefine them; for readers coming to the series from outside, each is restated in a single line where it first occurs. The senses of “object-local” and “positive structure” are given at their first occurrences, in §7.3 and §3.2 respectively.

## 1.3 Structure of This Paper

The core of this paper lies at a single point: dividing minimal extension into the two forms of instrumental extension and residual acknowledgement, and deriving from their difference the divergence between extensions that terminate and extensions that recur (§5.2–5.3). Sections 02–04 prepare the ground for this, §06 derives the corollary, and §07–08 verify the coordinates of the base case.

# 02 Three Strata of Non-Describability

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The word “non-describable” often gathers several distinct states of affairs under one heading. This paper divides them into three strata and makes explicit which stratum the imaginary dimension of this theory belongs to.

## 2.1 Epistemic Unknowability

The first stratum is non-describability arising from limits on the side of the knowing subject. An object cannot be grasped by present cognitive capacities, but might be grasped as those capacities develop — this stratum signifies not a limit in principle but a present epistemic shortfall. Since it becomes describable once cognitive capacity is extended, it is located potentially within the region of the real dimension. The imaginary dimension of this theory does not belong to this stratum. As Paper I §1.2 puts it: at the point where one can say “not yet known,” the matter has already entered the region of meaning as something knowable. Epistemic unknowability is a problem on the side of the real dimension.

## 2.2 Linguistic Ineffability

The second stratum is non-describability arising from limits on the expressive capacity of language. The tradition represented by Wittgenstein’s *Tractatus* (1922) — whereof one cannot speak, thereof one must be silent (7) — has made this stratum its theme. Mystical experience, aesthetic feeling and absolute alterity are located here. The imaginary dimension of this theory does not belong to this stratum either. Linguistic ineffability is a problem of the limits of

language as a medium of expression, whereas the imaginary dimension of this theory is a determination concerning the position of a structural aspect of the object itself (for detail, §7.2). Reference to the imaginary dimension does, however, itself demand a movement of meaning-formation, and in that instant the imaginary dimension comes to be located in the real dimension — a self-referential structure analogous to Wittgenstein’s “what can only be shown” (*Tractatus* 6.522). The analogy holds only at the level of the movement of showing versus saying; the imaginary dimension resides in the third stratum, treated next.

### 2.3 Structural Non-Describability

The third stratum is non-describability internal to the structure of the object itself. It is neither a limit of the knowing subject nor a limit of language: it means that a structural aspect of the object lies, for the operation of description, outside. The imaginary dimension of this theory belongs here. The tripartite definition of Paper I — pre-meaningful, non-manifest, non-defined — shows that the imaginary dimension is structurally required as the impossibility of closure of the real dimension. The imaginary dimension is the structural surplus of the operation of description, folded into the structure of the object itself. Structural non-describability cannot be dissolved by extending cognitive capacity or refining language. It is a structural property of the object itself, not a problem on the side of cognition or language. That the non-closure of this third stratum belongs to the structure of the real and not to an epistemic shortfall is taken up again in §3.1.

That this third stratum manifests not only in the structure of being but also in the structure of time was argued independently in Paper V. The coexistence of phases in the present is recovered neither by subdividing the real time axis  $t$  nor by replacing points with intervals (Muranushi, 2026e, §05) — this is neither a shortfall of cognitive capacity nor a limit of language, but the temporal manifestation of third-stratum non-describability: the structure of  $t$  as a descriptive axis cannot contain the coexistence of phases within itself. The third stratum is not a special circumstance localised to one region of ontology; it appears across descriptive systems generally — and this transversality is what motivates the formalisation of §05.

## 03 Methodological Foundations — Two Givens, Three Functions of Residue, Structural Avoidance

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This section sets out the methodological foundations of the theory: (a) the two givens this paper receives as granted, (b) why residue is necessary, and (c) what sort of operation structural avoidance is.

### 3.1 Two Givens — The Argument for Non-Closure and the Statement of Minimal Extension as a Principle

The argument that description fails to close — that description can capture only results already arisen as meaning, and necessarily leaves behind what lies prior to that arising — is borne not by this paper but by the response paper *The Far North of Real-Dimensionalism* (Muranushi, 2026d). By way of the retrospectivity of description and the regress argument — the argument that any operation seeking to bring the arising into description produces, from the point where it has been brought in, a further prior — that paper argued frontally, from the internal logic of Real-Dimensionalism, that what is left behind belongs not to an epistemic shortfall but to the structure of the enterprise of description itself. The series divides its labour at just this point: the argument for non-closure falls to 2026d, and the methodology for continuing to describe while carrying that non-closure falls to this paper. This paper does not re-prove non-closure; it receives it as given.

The impossibility of a closed system has been discussed as a structural analogy in several contexts, from Gödel’s incompleteness theorems to hermeneutic, structuralist and phenomenological traditions. What this paper receives as premise is a single point: that the real-dimension component of an object  $X$  does not by itself constitute a closed description (Axiom 1, non-closure, Paper I), and that this non-closure is not an accidental attribute of the object but a constitutive property (Paper I §4.1). The imaginary and real dimensions coexist within  $Z(X)$  as orthogonal components (Axiom 3, orthogonality), and this orthogonality means that reaching the imaginary dimension by quantitative extension of the real dimension is impossible in principle.

Paper V then established the mode of response to this non-closure as the principle of minimal extension: a description that fails to close requires an independent axis to complement it, and only the minimum needed to dissolve the

non-closure (Muranushi, 2026e, Principle 1). This paper's givens are therefore two: the argument for non-closure (2026d) and the statement of minimal extension as a principle (2026e). Its work lies at the confluence of the two — to show, as a single framework, what sort of operation it is to continue describing while carrying a non-closed description. The two points that follow — the three functions of residue (§3.2) and structural avoidance (§3.3) — are preparation for that.

### 3.2 The Three Functions of Residue — Derived from Three Relations of the Boundary

Where §3.1 received non-closure as given, here the question runs in the opposite direction: why is residue positively *necessary*? And why does that necessity appear as three functions, neither two nor four?

In the framework of this theory, the imaginary dimension as residue occupies the position of the closing boundary of the real dimension itself. The functions of residue are therefore derived from the structural relations a boundary bears to a system. This paper organises the structural correlates of a boundary into three — the boundary itself, the regions it divides, and the whole it delimits: a boundary relates (a) to itself, as something whose position may shift; (b) to the two regions it divides, as that which distinguishes inside from outside; and (c) to the whole it delimits, as something that may be folded back within that whole as its limit. The three functions of residue correspond one-to-one to this organisation.

First, from the mobility of the boundary is derived the function of *articulation*. Precisely because the boundary can move, the interior of the real dimension can be recomposed, making possible cognitive advance, theoretical development and the re-articulation of objects — this is what Axiom 5 (mobility of the boundary) of Paper I requires. A wholly closed system has no structural room for its boundary to move. Note that “closed system” here means not “a system without dynamic change of internal states” but “a system in which the distinction between inside and outside is itself unrecomposable.” It is only because the imaginary dimension is there as residue that the boundary of the real dimension can vary dynamically.

Second, from the differentiating operation of the boundary is derived the function of *identity*. This theory adopts a differential conception of identity (the Saussure–Derrida line): for it to be sayable that “this is X,” what is not X must lie outside the boundary. It is because the boundary divides inside from outside that the real dimension has identity as “what is not the imaginary dimension.” Since a wholly closed system has no outside, the identity of the system as a whole could not be determined.

Third, from the recursibility of the boundary is derived the function of *self-reflection*. The boundary can be folded back, within the whole it delimits, as the limit of that whole itself. By this folding, the description of the real dimension can display within itself the fact that it is not complete. What this theory requires is not a quasi-meta-standpoint internal to the system, but a structural meta-standpoint guaranteed by the system's non-closed component — the imaginary dimension as boundary.

On this organisation, the three functions are not an arbitrary enumeration by the author but are derived systematically from the three correlates of a boundary. Narrow the correlates to two and one of the functions drops out. A fourth correlate, on the other hand, does not arise from the boundary model adopted here: what a boundary can stand in relation to is either itself, or the parts it divides, or the whole it delimits, and this threefold division is mereologically exhaustive. The “three” of the three functions is thus neither an accident of enumeration nor a matter of taste in economy, but the minimal and exhaustive set of correlates independently required by the boundary model this paper adopts.

The “outside” invoked by the identity function is outside with respect to the real-dimension component D, not outside with respect to the totality  $Z = D + iD$ . The imaginary dimension is positioned outside relative to D while remaining immanent to Z (Axiom 2, immanence). The externality required by the identity function and the object-immanence this theory consistently stresses are compatible in virtue of this two-level distinction<sup>1</sup>.

### 3.3 Structural Avoidance — Distinguished from Negative Theology and the Philosophy of Silence

The operation this theory has adopted — never describing the imaginary dimension directly — differs from both negative theology and the philosophy of silence. What separates the three is not a difference of domain but the question of where non-describability is attributed. Negative theology (Dionysius, Eckhart, Cusanus and others) attributes it to the side of the object — a transcendent object that admits no direct predication — and approaches it by

accumulating negations. The philosophy of silence (Wittgenstein, the Zen tradition and others) attributes it to the limits of language or of representability, and makes no assertion about what lies beyond the limit. The structural avoidance of this theory is neither: it attributes non-describability to an object-immanent structural position relative to a descriptive system (§1.2, §8.7).

Structural avoidance therefore neither affirms nor denies transcendent reality. What it does is to carry out assertions about the object as structural description (meta-description from the side of the real dimension), and to fix their position as structural surplus. Since negative theology likewise proceeds by assertion, the difference between the two lies not in whether assertions are made but in what they are directed toward — toward a transcendent object, or toward the condition of description’s own non-closure. Structural avoidance too leaves a residue (§9.1), but that residue is not a concealed positive being; it is the structural surplus of the operation of description itself. This distinction by attribution is formalised in §05 as the distinction between instrumental extension and residual acknowledgement.

## 04 The Accumulation of Self-Limitation — Five Stages

The self-limitation of the preceding five papers has accumulated while changing form at each stage. Table 2 sets out the five stages; below, the details of each stage are left to the papers themselves and only the shape of the accumulation is traced.

[Table 2 Five stages of self-limitation]

| Stage | Form                   | Implementation                                                                                                                                                               |
|-------|------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| I     | Declaration            | “This paper does not describe the imaginary dimension itself directly” (Paper I §1.5)                                                                                        |
| II    | Methodology            | The determination “of imaginary-dimension origin” restricted to “retrospective description from the side of the real dimension, after capture” (Paper II §1.2, Definition 1) |
| III   | Formal apparatus       | $\Phi = P \times K \cdot V(Z) = \alpha D + \beta iD$ built into the formalism as real-dimension-side structural description (Paper III §1.4, §6.1)                           |
| IV    | Controlled suspension  | Only the thematisation of the opposing position suspended; the stratum “not arguing from the side of the imaginary dimension” retained (Paper IV §1.1)                       |
| V     | Statement as principle | The principle of minimal extension — “say no more than is required,” formalised at the level of theory construction (Paper V, Principle 1)                                   |

### 4.1 The Five Stages — From Declaration to Principle

The movement of the first three stages is, in a word, the internalisation of where the limitation resides — from a declaration placed outside the exposition (I), to a methodology governing the exposition as a whole (II), to the interior of a formal apparatus (III). The third stage marks the point of arrival. The observer function  $V(Z) = \alpha D + \beta iD$  retains the term  $\beta iD$  within the formalism while asserting no direct access to that term, describing structural connection to the imaginary dimension only by way of the sensitivity coefficient  $\beta$ . It is an implementation of structural avoidance built into the formalism.

At the fourth stage, the limitation becomes an object of operation. Paper IV declares at the outset that the preceding papers “have avoided treating the opposing position frontally. This paper suspends that self-limitation temporarily and thematises the opposing position. Its critical argument, however, relies solely on the internal logic of the opposing position and does not appeal to arguments from the side of the imaginary dimension” (Muranushi, 2026d, §1.1). What is suspended is only the stratum “do not thematise the opposing position”; the stratum “do not argue from the side of the imaginary dimension” is maintained throughout the suspension. Self-limitation is not a monolithic prohibition but an operable system whose strata can be designed for suspension or retention separately — and that was demonstrated here. If everything up to the third stage was the refinement of the limitation, the fourth stage is its operation.

And in Paper V, self-limitation ceased to be a particular declaration, method or apparatus and became the generative principle of theory construction: the principle of minimal extension. “Introduction is restricted to the minimum number of axes needed to dissolve the non-closure” (Muranushi, 2026e, Principle 1) — the spirit of structural

avoidance, “say no more than is required,” formalised at the general level of introducing descriptive axes in theory construction. That this statement as principle is carried through even into notational detail appears in verifiable form at one point: while adopting the complex notation  $T = t + it$ , that paper states explicitly that the algebraic property of the imaginary unit ( $i^2 = -1$ ) is “not used in any argument” (ibid., §04).

Declaration (I), methodology (II), formal apparatus (III), controlled suspension (IV), statement as principle (V) — across five stages, self-limitation grew from a particular asceticism, “do not describe the imaginary dimension directly,” into a general norm: description writes only what is required, and only at the level at which it is required.

## 4.2 The Sense of the Accumulation — The Condition of Entry

One insight can be drawn from these five stages: self-limitation has functioned not as a negative brake but as the condition enabling entry into deeper territory. The closer the territory entered came to a slide into metaphysical assertion — from ontology to phenomenology, to existential theory, to the bosom of the opposing position, and then to time — the deeper the level at which the limitation was built in: from declaration to methodology, to formal apparatus, to operation, to principle.

Across the five stages, the deepening of the limitation and the deepening of the entry run exactly parallel. This parallel is no accident: as §05 shows, self-limitation is the practical name of the theory of residue, and the depth of entry corresponds to how precisely the residue can be acknowledged. Self-limitation does not “demarcate the territory one may not enter”; it “supplies the structure required in order to enter.”

What this paper adds as a sixth stratum is not a further refinement of the accumulation. It is the extraction of the single grammar that has generated the accumulation as a whole — the theory of residue. This paper too inherits that self-limitation: it does not describe the imaginary dimension directly, but proceeds as a meta-description of its non-describability from the side of the real dimension.

# 05 The Theory of Residue — Two Forms of Minimal Extension and the Condition of Termination

## 5.1 Definition of Residue

Taking as given Definition 2 of Paper V (descriptive closure — when a descriptive system  $S$  can reconstruct the structure an object possesses without introducing a new independent axis irreducible to  $S$ ,  $S$  is closed with respect to that object) and Principle 1 (the principle of minimal extension), this paper defines residue as follows.

### [Definition A (Residue)]

When a descriptive system  $S$  is not closed with respect to an object, the principle of minimal extension requires an independent descriptive axis irreducible to  $S$ . Residue is the position, as seen from the side of  $S$ , of the structure that this required axis indicates. That is, residue denotes the structural position that does not appear within  $S$  but leaves its trace within  $S$  as  $S$ 's non-closure.

Three consequences follow immediately. First, residue is relative to  $S$ : to be residue is not an absolute determination but a position with respect to a particular descriptive system (a generalisation of the object-locality of §1.2). Second, residue is not a lack: it is what structurally guarantees the non-closure of  $S$ , and bears the three functions of §3.2 as boundary (a generalisation of positive structure). Third, the acknowledgement of residue — the operation of fixing residue in its position — is an operation from within  $S$ , not an operation that reaches the residue.

The three cases the series has treated are three satisfactions of this definition. Against the algebraic non-closure of the field of real numbers  $R$ : the imaginary unit  $i$ . Against the coexistence of phases on the real time axis  $t$ : imaginary time  $it$ . Against the descriptive non-closure of the real dimension  $D$ : the imaginary dimension  $iD$ . All three satisfy the same definition — yet, as the next subsection shows, they do not hold the same position<sup>2</sup>.

## 5.2 Two Forms of Minimal Extension — Instrumental Extension and Residual Acknowledgement

Set the three cases side by side and a fault line becomes visible inside their apparent isomorphism. The imaginary unit  $i$  and imaginary time  $it$  function, once introduced, as descriptive axes: complex numbers have positions on the

imaginary axis, and phases are located on the it axis. The imaginary dimension iD, by contrast, appears within no formal description at all, by the tripartite definition of Paper I (pre-meaningful, non-manifest, non-defined; Muranushi, 2026a, §1.2) — the development of this point is in Muranushi (2026d, §3.1). In respect of the describability of the introduced axis itself, the three cases fall into two groups. This paper names them as follows.

**[Definition B (Two forms of minimal extension)]**

Minimal extension, as a single class of operation, divides into two forms by the criterion of the describability of the introduced axis. The two forms are not, however, coordinate: within the framework of this paper, residual acknowledgement is taken as the basic form and instrumental extension is positioned as its terminable derivative (the ground of this non-coordination, and the limitation of the level at which it is claimed, are given at the end of §5.4).

(a) *Instrumental extension* — an extension in which the introduced axis itself becomes a describable object in the extended system. The addition of the imaginary unit  $i$  and the requirement of imaginary time it fall here. Upon the introduced axis, the structure of the object is located and described.

(b) *Residual acknowledgement* — an extension in which the introduced axis itself remains, even in the extended system, within structural non-describability (the third stratum of §2.3). The acknowledgement of the imaginary dimension iD falls here. What is fixed is not a position on the axis but the position that the axis is — the structural seat of the arising that description misses at every turn.

The difference between the two forms becomes visible if one compares what has become writable across the extension. What could not be written in the field of real numbers is the solution of  $x^2 = -1$ . Once the imaginary unit  $i$  is added, that solution can be written as “ $i$ ” — the added axis has become a stand on which objects rest. Since what could not be written has become writable, the very lack that required the addition has been filled. The same holds for imaginary time  $it$ , where phases are indicated as positions on the  $it$  axis. What cannot be written in the real dimension, by contrast, is what lies prior to arising as meaning. Once the imaginary dimension iD is acknowledged, this prior remains just as unwritable — what the acknowledgement fixes is the single fact that a structural seat is there, not a description of what occupies the seat. The iD axis does not become a stand; it remains on the side of the conditions of description.

The criterion of the distinction therefore reduces to one question: did the required axis become an instrument of description in the extended system, or did it remain a condition of description? In instrumental extension, the very lack that required the extension is filled. In residual acknowledgement it is not — not filling it is what acknowledgement means. That this difference divides the fate of the extension is set out as a proposition in §5.3 (this is the generalisation of Paper IV’s distinction, which called the positions of complex numbers in physics and of the imaginary dimension iD “diametrically opposed” — Muranushi, 2026d, §3.1). Hereafter, when this paper speaks of “whether the residue becomes describable,” it refers always to this one question, and the second half of the governing proposition is fixed in this sense.

It is worth pre-empting the question why the forms are exhausted by two. The criterion of the distinction — whether the extension axis itself becomes an object of description — is a two-valued question governed by excluded middle, and admits no third value. The conceivable intermediate case — an axis describable only in part — is not a third form but a mixture of the two. The describable part of the axis functions as an instrument, the non-describable part remains as acknowledged residue — and so long as the latter remains, Proposition P1b (§5.3) operates on that part. The bipartition is thus not an assumption about the simplicity of the cases but a consequence of the two-valuedness of the criterion.

This distinction corresponds exactly to the distinction by attribution confirmed in §3.3. Instrumental extension can be neutral about the reality of the axis — Paper V’s introduction of it as a “non-real descriptive axis,” without asserting its reality, is the correct self-understanding of instrumental extension. Residual acknowledgement likewise affirms no transcendent reality — but not by way of neutrality: rather, because the position acknowledged is by definition no descriptive content whatever.

### 5.3 The Condition of Termination — Why Mathematics Closes and Description Does Not

The distinction between the two forms makes it possible to state, in propositional form, an asymmetry the series has carried implicitly. Minimal extension in mathematics terminates: the field of complex numbers, obtained by adding  $i$  to the reals, is algebraically closed, and no algebraic equation any longer requires anything outside the field. The extension ends once and for all. Minimal extension in ontological description, by contrast, does not terminate: a description that acknowledges the imaginary dimension  $iD$  is itself a meta-description from the side of the real dimension, and that meta-description in turn leaves behind a further prior to arising (Axiom 4 of Paper I, the non-finality of description, and the regress argument of 2026d). When Paper V, discussing the three-stratum structure, reserved judgement by saying that it “does not assert ontological isomorphism,” what was being reserved was precisely this fault line.

This paper states the matter as the following propositions<sup>3</sup>.

#### [Proposition P1a (Necessary condition for termination)]

If a minimal extension brings a system to closure with respect to an object, then that extension is an instrumental extension. Contrapositively: an extension that is not instrumental — that is, residual acknowledgement, and mixtures containing it (§5.2) — does not bring the system to closure. The converse is not asserted: it is not claimed that an instrumental extension necessarily closes. Instrumental extension makes termination possible, by having the cause of the non-closure acquire a describable position on the introduced axis, but it does not guarantee it. The instance in which termination actually obtained is the algebraic closure of the field of complex numbers.

#### [Proposition P1b (Necessity of recurrence)]

Where the introduced axis itself remains within structural non-describability — that is, in the case of residual acknowledgement — the extension necessarily recurs. The description of the acknowledgement generates a further residue, and this recurrence has no terminus.

The conjunction of the two is referred to collectively as Proposition P1 (the condition of termination). P1b stands in a relation of strengthening to the contrapositive of P1a: residual acknowledgement not only fails to close (contrapositive of P1a) but positively recurs (P1b).

The word “recurrence” may be fixed once here. Recurrence in this paper is neither self-invocation in computer science nor inductive definition in mathematics. It denotes the iterative structure of an operation whereby acknowledgement becomes applicable once again to its own product — the description of the acknowledgement — and generates, at each application, a further residue. Where infinite regress often connotes a defect in argument, the recurrence of this paper is not an accusation of defect but a neutral structural description that the theory predicts of itself (the English term used is *recursivity*, to avoid confusion with *regress*).

The argument follows from the composition of the definitions of the two forms with the regress argument established in Paper IV. In instrumental extension, the structure that was the cause of the non-closure (for example, the square root of a negative number) acquires a describable position on the introduced axis — because the cause is dissolved, the extension can terminate (P1a). In residual acknowledgement, the cause of the non-closure lies in the structure of description itself: that what precedes arising is not contained in description. Acknowledgement does not dissolve this cause — not dissolving it is precisely what acknowledgement is defined as. The moment the acknowledgement is written down, what is written down is one arisen result, and its prior withdraws still further (the regress argument). Hence it recurs (P1b). What should be made explicit here is the dependency structure of P1b: P1b follows not from Definition B alone, but from the composition of the definition of residual acknowledgement with the regress argument established in Paper IV (Muranushi, 2026d, §04). Definition B yields only “acknowledgement does not dissolve the cause”; the driver of the recurrence — that an undissolved cause generates a further residue at each acknowledgement — is borne by the regress argument.

Two corollaries follow from these propositions.

#### [Corollary 1]

“The impossibility of a closed system” is a corollary of Proposition P1b: since the extension required by the description of the real dimension is residual acknowledgement, no extension brings the system to closure. The impossibility of a closed system acquires its position not as an independent axiom but as a consequence of the fact that the required residue cannot become describable (its residence in the third stratum).

#### [Corollary 2]

The meta-level limitation of this paper itself (§9.1) is likewise a self-application of Proposition P1b. This paper has described the theory of residue, but that description is itself an acknowledgement, and thus constitutes one turn of the recurrence. A theory contains its own incompleteness as a proposition, and that proposition applies to itself — that this self-application does not break down shows that the operation of residual acknowledgement is internally coherent. It is not, however, evidence of validity as seen from outside the theory: what is claimed here is consistency, not correctness.

### 5.4 The Describability of Imaginary Time $it$ and the Imaginary Dimension $iD$ — Resolving a Tension Internal to the Series

The distinction between the two forms resolves a tension the series has carried since Paper V. The imaginary time  $it$  of Paper V functions as a descriptive axis: phases are located upon it and described. The imaginary dimension  $iD$  of Paper IV, by contrast, appears within no formal description. The two share the prefix “imaginary,” yet their describability looks diametrically opposed. Is imaginary time not the temporal version of the imaginary dimension — and if it is, why can one be described and not the other?

The resolution has two steps. First, imaginary time  $it$  is an instrumental extension and the imaginary dimension  $iD$  is a residual acknowledgement: the two belong to different forms of Definition B, and the difference between them is not confusion but classification. What the prefix “imaginary” shares is only the formal position of “an independent axis orthogonal to the axis of the real component”; describability may differ from form to form.

Second — and this is the substantive resolution — locating phases on the  $it$  axis is the temporal version of the operation established by Definition 1 of Paper II (the determination “of imaginary-dimension origin” is retrospective description from the side of the real dimension, after capture). The  $it$  axis is thus in itself a descriptive apparatus on the side of the real dimension, and the ground on which that apparatus is required — the very working by which phases arise at each moment — is surplus of the  $iD$  type. The apparatus is describable; the arising that required the apparatus is not. Paper V’s reservation that it “does not assert the reality of it” was the correct operation of these two levels, and the present subsection merely gives that operation the name of a distinction between two forms.

From this analysis of imaginary time  $it$ , the ground of the non-coordination announced in Definition B is generalised: no instrumental extension is required unless the system’s failing to close precedes it. Instrumental extension is a response to non-closure, not the cause of non-closure. In this order, residual acknowledgement is basic and instrumental extension is positioned as its terminable derivative. Even the addition of the imaginary unit  $i$  was a response to the state of affairs in which the number system failed to close.

The level at which this positioning is claimed should, however, be made explicit. What is claimed here is confined to isomorphism at the level of Definition 2 (descriptive closure) — that each extension is a response to one and the same state of affairs, namely that the structure of the object cannot be reconstructed without an independent descriptive axis. Reducing the algebraic non-closure of the field of real numbers to the third-stratum non-closure by which description misses the arising (§2.3) is not a claim of this paper. Such a reduction is excluded by the same discipline as in §6.3 — do not attribute to a system whose antecedent has not been argued the structure of a system for which it has. The ordering of basic and derivative is a positioning internal to the present framework at this limited level, not an ontological interpretation of mathematical non-closure.

## 06 Structural Simultaneity — Derived from the Theory of Residue

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Paper V, in its conclusion, treated the coexistence of phases in the present as “a special case of structural simultaneity in which the separation is very small,” and left a general theory of it as a task for subsequent work. This section takes up that task not as an independent theory but as a corollary of the framework of §05.

### 6.1 Definition

#### [Definition C (Structural simultaneity)]

When a descriptive system  $S$  possesses an order structure (before and after, greater and lesser, seriality), and structures at two or more positions separated in the order of  $S$  belong together in a single structural field on a descriptive axis orthogonal to  $S$  — this is called structural simultaneity. The belonging-together is irreducible to the order relations of  $S$  (were it reducible, the orthogonal axis would not be required), and from the side of  $S$  it appears as a coexistence that the description of  $S$  cannot recover — that is, as the trace of a residue. “Structural field” denotes a descriptive position that preserves a belonging-together inexpressible in the order relations of  $S$ , and implies no region of the real whatever.

The thickness of the present argued for in Paper V is, on this definition, the case where  $S = t$  (the real time axis) and the separation is minute. The scope of Definition C is wider than that — in the magnitude of the separation, and in the kind of descriptive system.

### 6.2 Belonging-Together of Phases Widely Separated on the $t$ Axis

One example of structural simultaneity with a large separation: an event of decades ago belongs to the present not as recollection — as reference to a past position on the  $t$  axis — but as an internal articulation of what is now arising. The criterion separating reference from belonging-together is supplied directly by the argument of Paper V: reference is a relation between two positions on  $t$ , whereas belonging-together is a relation within a single present, and the two differ in category (Muranushi, 2026e, §05). The magnitude of the separation adds nothing to this categorial distinction. Indeed, no stage of Paper V’s argument — that subdivision produces no thickness, that relations of before and after do not convert into belonging-together, that the folding of retention cannot eliminate the indexical standpoint — presupposes minuteness of separation. No new argument is therefore needed for this widening; confirming the range of application of the existing argument suffices.

### 6.3 Range of Application and Limitation of Scope

Definition C does not restrict  $S$  to  $t$ . For any descriptive system with an order structure — a series of logical priority, a causal series, the linearity of a text, the number series — the same form operates if a belonging-together irreducible to that order is confirmed. This paper does not, however, assert the existence of such belonging-together in each such system. What it presents is a conditional: if there is a belonging-together that the order of  $S$  cannot recover, then the principle of minimal extension requires an orthogonal axis, and the belonging-together is located as structural simultaneity on that axis. Establishing the antecedent is a matter for individual argument system by system, and Paper V accomplished it for  $S = t$ .

What this paper maintains at the level of claim is only three points: (a) structural simultaneity for  $S = t$  (argued in Paper V); (b) that this argument does not depend on the magnitude of the separation (§6.2); and (c) the form as a conditional (Definition C). Reference to systems other than  $t$  remains an indication of applicability. This limitation is not a negative reservation but an application of Norm 1: to attribute the existence of belonging-together to a system whose antecedent has not been argued would be to introduce a structure required by no structure whatever.

### 6.4 Structural Integration with the Concept of Occurrence

The second task set by Paper V — “structural integration with the concept of occurrence in Paper II” — is discharged in the same way. Occurrence was the state of affairs in which a fine oscillation of imaginary-dimension origin is captured in cognitive space (Muranushi, 2026b). In the vocabulary of structural simultaneity: the capture of an occurrence is the retrospective description, from the side of the real dimension, of a belonging-together in the it

direction, in which something that does not yet have a position on the *t* axis (something in the course of arising) cuts into the stratum of the present. That an occurrence can be captured as “prior to actualisation” is only because the present has a stratified structure in the *it* direction — a present as a point without width offers no place into which the prior might cut. Conversely, the arising of phases, which was the ground on which the *it* axis is required, is in the vocabulary of Paper II the stratum in which occurrences take place. The two papers were describing one and the same structure, from the side of temporal description (V) and from the side of phenomenological capture (II); integration is not the construction of a new theory but the confirmation of this identity.

## 07 A Genealogy of Philosophical Surplus Concepts — Continental, Eastern, Analytic, Indexical

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What is measured from this section onward is not the abstract form of the present theory but its base case — the imaginary dimension. Since instrumental extension is a derivative of residual acknowledgement (§5.4), fixing the base case on the coordinates of the history of philosophy constitutes the substance of testing the framework. Below, surplus concepts are organised as structural analogues of the imaginary dimension, drawn from four genealogies: Continental, Eastern, analytic and indexical. Settling the interpretation of each thinker is left to specialist scholarship; what is undertaken here is the organisation of structural differences as seen from the present framework.

### 7.1 Kant’s *Ding an sich*

Kant’s *Ding an sich* is isomorphic with the imaginary dimension in that it cannot be known directly yet is required as the structure that makes knowledge possible. The decisive difference lies in its location: whereas the *Ding an sich* is positioned outside the knowing subject, the imaginary dimension of this theory is immanent to the object itself (Axiom 2, immanence). The imaginary dimension shares unknowability and does not share externality with respect to a subject (for detail, §8.6).

### 7.2 Wittgenstein’s “What Cannot Be Said”

Wittgenstein’s *Tractatus* (1922) distinguishes what can be said (within the limits of language) from what cannot (beyond the limit), and concludes that of the latter one must be silent. Ethics, beauty and the religious belong here. “What cannot be said” and the imaginary dimension are isomorphic in avoiding direct verbalisation, but their attribution differs (§3.3): the former is attributed to the limits of language, the latter to the structure of the object itself. In Wittgenstein’s framework the object is located on the sayable side and what cannot be said is placed beyond it. The imaginary dimension is not beyond, but a surplus within the structure of the object.

### 7.3 Heidegger’s *Ereignis* / *Lichtung*

Later Heidegger’s *Ereignis* and *Lichtung* treat the structure of Being (*Sein*) as the condition on which beings (*Seiende*) appear as beings. *Ereignis* / *Lichtung* and the imaginary dimension run parallel in positioning something as the structure conditioning the appearance of beings (the real dimension), which cannot be grasped directly from the side of those beings. Heidegger’s *Ereignis*, however, frequently connotes an ontological or cosmological primordality. The imaginary dimension of this paper, by contrast, is determined object-locally, as structural surplus relative to a particular object *X*, and is not treated as a cosmic principle — for in the framework of this theory the cosmos is a totality already articulated as an object of description, and thus belongs to the side of the real dimension.

### 7.4 Lacan’s *Réal*

Lacan’s *Réal* is the impossible, ungraspable from the symbolic, and runs parallel with the imaginary dimension as “the surplus guaranteeing the impossibility of closure of the system of meaning (the real dimension).” Whereas the *Réal* is bound up with desire and trauma within a psychoanalytic theory of the subject, the imaginary dimension of this theory is structural description in the general domain of objects and carries no implications for a theory of the subject (§8.5).

## 7.5 Derrida's *différance* / trace

Derrida's *différance* (lecture 1968, collected 1972), which shows that the appearing of the here and now is constituted only through difference from others and deferral, is the closest of the concepts treated here to the imaginary dimension. The two nevertheless belong to different logical categories: *différance* is an operational concept denoting a movement, while the imaginary dimension is a structural position (§8.4).

## 7.6 Deleuze's *virtuel*

Deleuze's *virtuel* (*Difference and Repetition*, 1968) is positioned as the latent, generative ground of *actuel* being, and runs parallel with the imaginary dimension in that the *actuel* is constituted without exhausting the *virtuel*. Whereas the *virtuel* is a dynamic, intensive structure of genesis, this paper determines the imaginary dimension as a structural position rather than a movement of genesis. This is not a denial of dynamism: the dynamism of real-dimension-side operations — the mobility of the boundary (Axiom 5), the arising of occurrences (Paper II), the dynamic maintenance of  $\Phi = P \times K$  (Paper III) — is positively thematised, and it is only the imaginary dimension itself that is determined as a structural position.

## 7.7 Adjacent Continental Genealogies

Around these six concepts lies a family of notions overlapping topologically with the imaginary dimension, beginning with Blanchot's *dehors*. Agamben's *reste* in particular resonates directly, in its very term, with this paper's concept of residue, but its problematic is messianic and political-philosophical, and its domain of application differs.

## 7.8 Genealogies of Surplus in Eastern Thought

In parallel with the Continental line, Eastern thought too has a long genealogy of "what cannot be described directly yet is structurally required." The concepts below all run parallel with the imaginary dimension of this theory in avoiding direct positive assertion, refusing reduction to existing categories, and being required as conditions of structural constitution. On the other hand, the imaginary dimension of this theory is pure structural description without the soteriological or practical implications many of these Eastern surpluses carry, and is an object-local surplus — and on these counts it is consistently distinguished from the genealogy as a whole. Taking this shared difference as read, only the points specific to each concept are stated below.

*Śūnyatā* in Nāgārjuna — the *Mūlamadhyamakakārikā* (c. 2nd c.) argues that all existents lack own-being (*svabhāva*, independent essence). Emptiness is not nihility but is positioned as a determinative characteristic, that of possessing no independent substance, and shares structure most broadly with the imaginary dimension of this theory within this genealogy.

*Ālaya-vijñāna* in Yogācāra — in the eight-consciousness doctrine of Vasubandhu and others, this is determined as the deepest consciousness, a substratum storing the seeds (*bīja*) of all phenomena. Here lies the most important distinction within this genealogy. Seen from the framework of this theory, the *ālaya-vijñāna* belongs to the depths of the real dimension, as an information space that retains and circulates all the information of phenomena. The imaginary dimension of this theory, by contrast, is a structural surplus prior to information and prior to meaning, and is not a stratum that holds information. The two belong to fundamentally different strata — "information space (the depths of the real dimension)" and "structural surplus prior to information (the imaginary dimension)" — and must not be conflated.

Kūkai's ontology of language — the *Shōji jissōgi* (early 9th c.) holds that language (voice and letter) does not serve as a medium of description but constitutes the structure of reality (*jissō*) itself. It runs parallel with the imaginary dimension of this theory in positing an unarticulated stratum that requires articulation into the surface, and in the impossibility of direct access to that stratum.

Nishida Kitarō's logic of place — the pure experience of *An Inquiry into the Good* (1911) as direct experience prior to the separation of subject and object, and the later logic of place as the place of absolute nothingness, show that the "place" that constitutes beings cannot itself be objectified as place. It runs parallel with the imaginary dimension of this theory in the structure prior to objectification and its unobjectifiability. Whereas Nishida's place is developed as a structure of consciousness, however, the imaginary dimension of this theory is a structure of the object X.

Izutsu Toshihiko's theory of the linguistic ālaya-consciousness (*Consciousness and Essence*, 1983; *Toward the Depths of Meaning*, 1985) is an ontology of language that integrates Yogācāra with Kūkai while requiring a stratum of "meaning before meaning," and is the closest Eastern point of reference to the imaginary dimension of this theory, in the non-describability of the prior stratum and its structural requirement from the surface.

As adjacent genealogies, the non-action (*wuwei*) and the Way (*dao*) of Laozi and Zhuangzi, the Zen formulas of not standing on words and of transmission outside the teachings, and the Huayan realm of the unobstructed interpenetration of phenomena all belong to this Eastern tradition.

## 7.9 Surplus in Analytic Philosophy — Remainder as Intrinsic Content

Alongside the Continental and Eastern lines, twentieth-century Anglophone analytic philosophy also has a genealogy in which formal or structural description necessarily leaves part of reality behind. The knowledge argument (Jackson, 1982) holds that felt quality is left behind however complete physical and functional description may be; the Newman problem for structural realism (Newman, 1928; Demopoulos & Friedman, 1985) shows that description of structure alone does not uniquely fix the intrinsic content of an object. Russellian monism (Russell, 1927; Stoljar, 2006; Chalmers, 2010) holds that what physics describes is confined to structural and relational properties, while the intrinsic, categorical properties that fill that structure remain outside description; and neutral monism (Russell, 1921) posits a neutral substrate, neither mental nor physical, prior to both.

These analytic surpluses share the direction "formal description necessarily leaves part of reality behind," but differ decisively from the imaginary dimension of this theory in what is left behind. What the analytic line points to is the intrinsic, categorical content that fills the structure — a stratum that stands statically there while not appearing in structural description. The imaginary dimension of this theory, by contrast, concerns not content but genesis. What is left behind is not "what fills the structure" but "how that articulation came to arise"; where the former is a static gap between structure and content, what this paper takes aim at is the genetic gap between description and arising — the position of the arising that description misses at every turn. The argumentative development of this content/genesis distinction is in *The Far North of Real-Dimensionalism* (Muranushi, 2026d, §4.5); this section uses it as a coordinate for organising the genealogy.

## 7.10 The Indexical Residue — Consequences from Perry and Mellor

Within the analytic surplus there is a second vein, distinct from the genealogy of the preceding subsection: the debate over essential indexicals. Perry (1979) showed that the indexical "I," "now" and "here" are not exhausted by any non-indexical description — dates, coordinates, proper names. Paper V refined the scene in which this residue appears in the structure of time: Mellor's (1998) token-reflexive account supplies truth conditions for sentences containing "now," but does not supply the stratified structure of the present. When the reduction to truth conditions is complete, what has been left behind is the internally articulated stratum itself, upon which phases occupy positions of greater or lesser depth (Muranushi, 2026e, §05).

What makes this vein important for organising the genealogy is the character of the residue left behind there. Where the remainder of qualia and Russellian monism is a surplus of content — a static substrate that fills the structure while not appearing in structural description — the indexical residue is not content. "Now" is neither substrate nor property; it marks the site of arising. It is a residue on the side of genesis, escaping at every turn the coverage of description, and in the content/genesis division established at 2026d §4.5 it is, so far as this paper has ascertained, the only case in the analytic line that aligns on the genetic side. Moreover, Paper V's argument showed that this residue does not suffice as a mere marker but requires an axis of its own — the indexical residue is a vein that had reached, from within analytic philosophy, as far as the requirement of minimal extension. Where Paper IV took the independent convergence of three inquiries of differing provenance — passive synthesis, rule-following, contingency — as a sign of non-closure (Muranushi, 2026d, §4.4), this adds a fourth testimony to that convergence, and a testimony from the analytic metaphysics of time at that.

Within the same analytic metaphysics there is a counterpart differing in attribution. Fine's fragmentalism (Fine, 2005), referred to in Paper V, responds to the very same phenomenon of the coexistence of phases by fragmenting reality itself: coexistence, on that view, is a structure on the side of the real. The structural avoidance of this paper stands on exactly the opposite side, treating coexistence as a state of affairs on the side of description and reserving

judgement about the real (§3.3). Faced with one and the same phenomenon, it is where non-describability is attributed that divides the two.

## 08 Five Readings of the Imaginary Dimension and the Position of This Paper

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Building on the genealogical survey of §07, this section takes the tripartite definition of the imaginary dimension (pre-meaningful, non-manifest, non-defined) as a coordinate system and clarifies which aspect of the imaginary dimension each existing concept captures and which it misses. The order matters: existing concepts are not the starting point from which the imaginary dimension is classified; the tripartite definition is the starting point by which existing concepts are measured.

### 8.1 The Tripartite Definition of the Imaginary Dimension

According to Paper I §1.2, the imaginary dimension is determined by the following tripartite definition. (a) *Pre-meaningful* — the state prior to arising as meaning. (b) *Non-manifest* — the state of not having taken a form that appears in the real dimension. (c) *Non-defined* — the state of not being determined as an object located within a framework. The imaginary dimension of this theory is a position satisfying all three simultaneously, and any reading capturing only one of them misses its determinate position. Note that “pre-meaningful” here means not *meaningless* but prior to semantic articulation. Likewise “non-manifest” and “non-defined” denote not a negative nothingness but, respectively, a stratum preceding manifestation and a stratum preceding definition.

### 8.2 The Imaginary Dimension Read as Potentiality

The genealogy of potentiality, originating in Aristotle’s *dynamis*, offers the possibility of reading the imaginary dimension as “what has not yet arisen but might arise.” Potentiality captures the non-manifestness of the imaginary dimension but misses the pre-meaningful. Potentiality is already invested with meaning as a potency having some directedness (a seed, for instance, is the potency of a tree). The imaginary dimension, by contrast, carries no directedness of meaning-formation whatever. The potentiality reading is a partial reading, capturing only one of the three terms of the definition.

### 8.3 The Imaginary Dimension Read as Absence

The concept of absence, developed in the line running through Sartre and Blanchot, offers the possibility of reading the imaginary dimension as a negative mode of being in which what ought to be is not. Absence captures the non-manifestness of the imaginary dimension negatively, but this is a denial of positive structure. The imaginary dimension of this theory is not mere absence but a positive structural remainder. Since absence is always absence of something, it also already presupposes some meaning, and so fails to capture the pre-meaningful.

### 8.4 The Imaginary Dimension Read as Différance

Derrida’s *différance* satisfies the pre-meaningful and the non-manifest, and captures the non-defined in part — it is the closest of the five readings. As argued in §7.5, however, *différance* is an operational concept and the imaginary dimension a structural position; the two belong to different logical categories.

### 8.5 The Imaginary Dimension Read as Unconscious Reserve

The concept of an unconscious reserve, developed in the psychoanalytic tradition of Freud and Lacan, offers the possibility of reading the imaginary dimension as an accumulation that does not appear at the surface of consciousness but is located in the internal structure of the subject. Unconscious reserve captures the non-manifestness of the imaginary dimension but misses the pre-meaningful: an unconscious reserve already belongs to the region of meaning, as repressed meaning, displaced meaning, desire. Moreover, the imaginary dimension is not an internal structure of the subject but a structural remainder of the object. The frameworks to which the two apply differ fundamentally.

### 8.6 The Imaginary Dimension Read as Transcendental Outside

The concept of a transcendental outside, developed in the line of Kant’s *Ding an sich*, offers the possibility of reading the imaginary dimension as “what cannot be grasped by cognitive capacities yet is required as the structure making

cognition possible.” The transcendental outside captures many aspects of the tripartite definition. As argued in §7.1, however, Kant’s transcendental outside is located outside the knowing subject, whereas the imaginary dimension of this theory is immanent to the object itself. The character of the “outside” differs essentially.

### 8.7 The Imaginary Dimension of This Paper — As Structural Surplus

As the examinations of §8.2–8.6 show, each of the five readings captures only part of the tripartite definition. A reading that satisfies all three simultaneously while preserving object-immanence is not found within the comparative range of this paper.

What has been shown here is not the superiority of the imaginary dimension over the five readings but the distinguishability of its position: the tripartite definition is the coordinate system of the present framework itself, and the finding that each reading misses one of its terms shows only that the imaginary dimension occupies a structural position distinct from every existing concept. The Eastern (§7.8) and analytic (§7.9) concepts likewise resonate deeply with the imaginary dimension of this theory while possessing scopes of their own.

In summary, the imaginary dimension as “structural surplus” in this theory acquires its contour through the combination of six determinations: (a) it is immanent to the object; (b) it is a positive structure; (c) it is prior to meaning-formation; (d) it is determined as a structural position within a descriptive framework; (e) it carries no soteriological, practical or personal implications; and (f) it is an object of residual acknowledgement — the acknowledgement of the imaginary dimension is not an instrumental extension (§5.2), the imaginary dimension does not come to stand on the side of descriptive content even after acknowledgement, and hence that acknowledgement recurs (Proposition P1b). Where (a) to (e) contour “what the imaginary dimension is,” (f) contours “what sort of operation it is to write the imaginary dimension down” — and with the addition of an operational determination to determinations of being, the coordinate system is complete. What this paper presents is not superiority but the describability of overlaps and differences with the family of existing concepts; placement within the history of philosophy is itself left to future comparative research.

## 09 Discussion and Limitations

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### 9.1 The Meta-Level Structure — From Confession to Proposition

This paper has a self-referential structure: to say “the imaginary dimension is non-describable” is itself an operation upon the imaginary dimension from the side of the real dimension, and this meta-description itself carries a descriptive surplus. Formerly the framework had no option but to confess this as a limit descending upon the theory from outside. In this paper it becomes instead a self-application of a proposition the theory derives about itself — residual acknowledgement recurs (Proposition P1b) — to this paper itself (Corollary 2, §5.3). The confession of a limit is thereby recovered within the theory — including, of course, the fact that the text recovering it generates a further residue.

### 9.2 Open Problems

The following remain as open problems specific to this paper. (a) Refinement of the mutual relations among the three terms of the tripartite definition of the imaginary dimension (pre-meaningful, non-manifest, non-defined). (b) Refinement, in concrete cases, of the decomposition of axes describable only in part — mixtures of the two forms (§5.2). (c) System-by-system treatment of whether the antecedent of Definition C (belonging-together irreducible to order) obtains in systems other than the real time axis. (d) The task left by Paper V: whether the algebraic property of the imaginary unit (the rotation fixed by  $i^2 = -1$ ) can operate in formalising transitions between phases. Empirical connection in domains of application lies outside the scope of this paper.

## 10 Conclusion

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Any enterprise that sets out to describe the world meets, at some point, a wall: from here on, this cannot be described directly. In most cases thought ends there in one of two ways — halting before the wall, or fleeing into the mystical. Either way, description ends. What this series has demonstrated across six papers is a third way: relocating the wall as

“the structural surplus of description,” building it in not as the outside of description but as what makes description possible, and so continuing to describe while carrying the wall. This paper has, for the first time, developed that way into a single theory.

The conclusion is recovered by the governing proposition stated at the outset: a descriptive system can be extended beyond itself only in virtue of failing to close, and whether that extension terminates or recurs is settled by whether the required residue itself becomes describable. The first half is a consequence of Definition 2 and Principle 1 of Paper V; the second half is this paper’s contribution. That is: minimal extension divides, by the criterion of the describability of the introduced axis, into instrumental extension and residual acknowledgement (Definition B), and only the former can bring a system to closure, while the latter necessarily recurs (Proposition P1). The three operations the series has performed separately — the addition of the imaginary unit  $i$ , the requirement of imaginary time  $it$ , the acknowledgement of the imaginary dimension  $iD$  — are unified under this single classification, and the accumulated self-limitation of the preceding five papers (§04) has been redescribed as the practical implementation of this theory. Structural simultaneity (Definition C) has been given as a corollary of the theory, without requiring an independent theoretical construction.

Compressed into one sentence, the finding of this paper is this: that extension in mathematics terminates at a stroke while description of being continues without end is not a difference in the capacity of those who describe, but a difference in whether the required residue becomes describable. And residue is not the name of what description has dropped; it is the name of the very structural position by way of which alone a descriptive system can exceed itself. This is the position to which the title “the ontology of residue” points.

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The field of real numbers met a single state of affairs — that it is not algebraically closed — added a single axis, and closed. The description of being meets an isomorphic state of affairs anew each time it writes. What algebra performed once and completed, the description of being continues without end — and that endlessness is not a defect but the name of the exact distance the enterprise of description holds to the arising of the real. The ontology of residue is nothing other than the name of a discipline that keeps describing this distance as structure.

## Notes

- 1 What this section derives from the boundary is the three functions of residue, not the claim that this residue has the particular content of an imaginary dimension satisfying the tripartite definition (pre-meaningful, non-manifest, non-defined). Deriving the functions and determining the content are distinct matters; the latter was established constructively by the preceding papers. This section presupposes that determination of content.
- 2 The applicability of Definition A is not confined to the three cases of this series. As a structural analogue one may cite the codified legal system with the open texture described by Hart (1961). A legal system that cannot fix in advance, by enumeration of rules, the whole of their application responds to that non-closure through general clauses such as good faith and public order — descriptive axes independent of and irreducible to enumeration. Whether this analogue belongs to one or the other form of Definition B, and whether its antecedent strictly obtains, lie outside the scope of this paper.
- 3 They are called “propositions” rather than “theorems” because what follows is neither a mathematical theorem proved within a formal system nor a theorem of logic, but a philosophical proposition following meta-theoretically from Definitions A and B — to give a name exceeding the strength of the argument would itself offend against the spirit of Norm 1.

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