Imaginary Time

— Descriptive Non-Closure and the Principle of Minimal Extension —

虚時間 —— 記述の非閉鎖性と最小拡張原理 ——

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ï.Fragmenta — Fragments from the Imaginary Dimension

Abstract

This paper sets out from a single circumstance: the description of what we ordinarily call "time"—the one-dimensional flow taken to run in one direction from past to future, the time axis t—does not close on its own. A description that does not close demands independent descriptive axes that complement it, and demands them only minimally. This paper names this general policy the Principle of Minimal Extension and applies it to the structure of time. As its consequence, time is described as complex time T = t + it, given as the composition of the real time axis t and the imaginary time axis it orthogonal to it. This unfolds into the structure of time the ontological formula presented in the first paper, "Extended Imaginary Number Theory: A Conceptual Framework for the Dual Structure of Existence" (Muranushi, 2026a), and corresponds structurally to the historical extension of the number system from the one-dimensional number line to the two-dimensional complex plane.

The central claim of this paper condenses into one point. The present is a field in which different phases coexist. Onto the phase now occurring, the phase just passing away and the phase about to arrive are superimposed—and this coexistence of phases is not reducible to the before-and-after relations of t: it is not recovered however finely the real time axis t is divided, nor by replacing it with any interval. That phases separated on t coexist within a single present—this structural simultaneity—can be placed only on an independent descriptive axis orthogonal to t. This irreducibility is the paper's central ground for demanding it as an independent descriptive axis.

Unlike imaginary time in physics (the Wick rotation; the Hartle–Hawking no-boundary proposal), which is introduced technically as an analytic continuation of real time, the imaginary time it of this paper is demanded as a descriptive axis independent of t. The two do not compete; they are orthogonal at different descriptive strata. This paper does not assert the reality of imaginary time it; it addresses the descriptive circumstance that the description of real time t does not close on its own.

Keywords: imaginary time; structural simultaneity; coexistence of phases; descriptive non-closure; principle of minimal extension; two-dimensional structure of time; complex time; irreducibility; indexical present; projection; phenomenological time; hypertime; Hartle–Hawking; extended imaginary number theory

Contents

01 Introduction — From the Non-Closure of Linear Time to Minimal Extension

Number was once a one-dimensional line. Square roots of negative numbers were held to be impossible, and numbers were represented as points on the number line. From the sixteenth century onward, however, imaginary numbers appeared again and again in the course of calculation, and were eventually formalized as an independent axis orthogonal to the real axis—as one coordinate of the two-dimensional complex plane. From a one-dimensional line to a two-dimensional plane: this extension widened the very structure of mathematics from its foundations and carried decisive implications for domains as wide-ranging as quantum mechanics, electromagnetism, and signal processing.

This paper proposes a structurally isomorphic extension for the description of time. Its point of departure, however, is not the conception of two-dimensional time but a single insufficiency harbored by the one-dimensional description of time. The description of the time we ordinarily experience—t, a one-dimensional flow running in one direction from past to future—cannot contain within itself the phases that coexist in the present. When, against this non-closure (detailed in §05), complementary independent axes are added only minimally (the Principle of Minimal Extension, §02), the adequate description of time is given as the composition:

$$T = t + it$$

Here t is the real time axis, it is the imaginary-time-axis component orthogonal to it, and T is complex time given as the composition of the two. Just as, on the complex plane, the real and imaginary axes are orthogonal and a complex number $z = a + bi$ is given as a point on that plane, so the t axis and the it axis are orthogonal, and complex time T is described as a structure on that temporal plane.

This claim is not the mere addition of a vocabulary item. The categories we call self, causality, and history are constructions that first become possible on the premise that time is a one-dimensional line; once time is described as a two-dimensional structure, they are relativized as one-dimensional projections of the two-dimensional structure T.

The two-dimensional form is introduced not as a premise but only as a consequence of non-closure. The principal aim of this paper lies not in the claim that time is two-dimensional as such, but in placing the coexistence of phases in the present—not inside the before-and-after relations of punctual time, but as a phase structure orthogonal to them. Complex time $T = t + it$ is the form for placing this overlap of phases.

Before entering the argument, one term must be fixed. What this paper calls a phase is neither a topology in the mathematical sense nor the phase of a wave in the physical sense. It denotes an experiential facet constituting the present—that which is now occurring, that which is just passing away, that which is about to arrive—and the word is used in this sense only.

In what follows, Section 2 presents the tiered structure spanning number, dimension, and time and demarcates the thematic position of this paper. Section 3 situates the paper within the physical and philosophical lineages of the concept of imaginary time; Section 4 gives the formal specification of the core formula together with the projectivity of real time t. Section 5 discusses the proper domain described by the it axis; Section 6 makes explicit the orthogonal relation to the Hartle–Hawking no-boundary proposal; and Section 7 concludes.

02 The Tiered Structure — Number, Dimension, Time

This paper belongs to the framework of the Extended Imaginary Number Theory series (Muranushi, 2026a, 2026b) and unfolds its tiered structure into the structure of time. The ontology of the series as a whole is organized as the following three-tier structure, grounded in the symmetric pairing of a real and an imaginary component.

> **[Definition 1 (Three-Tier Structure)]**
>
> First tier—real and imaginary numbers in the domain of number. Second tier—the real dimension D and the imaginary dimension iD in the structure of dimensions. Third tier—real time t and imaginary time it in the structure of time. Each tier possesses a structure formally corresponding to its neighboring tiers and unfolds into its corresponding domain.

The first tier is mathematical fact. In a complex number $z = a + bi$, a gives the coordinate on the real axis and bi the coordinate on the imaginary axis orthogonal to it. The two are structurally orthogonal, and any point of the complex plane is uniquely given by this orthogonal decomposition.

The second tier is the thesis of the first paper (Muranushi, 2026a). The totality Z of an object's existence is described as the superposition $Z = D + iD$ of the real-dimensional component D, which appears in description through observation, articulation, and conceptualization, and the imaginary-dimensional component iD, the structural surplus that description does not exhaust.

The third tier is the subject of this paper. The structure of time is given, under a symmetry of the same form as the second tier, as the composition of the real time axis t and the imaginary time axis it. As the second tier was the unfolding into the structure of dimensions, so the third tier is the unfolding into the structure of time; the two stand in a formally isomorphic relation.

This paper does not, be it noted, assert an ontological isomorphism among these three tiers. The correspondence of the tiers is an ordering of the fact that one and the same formal structure recurs across different domains, and is to be understood as structural correspondence.

What sustains this recurrence is a single descriptive mechanism common to the tiers. To specify it, we first define descriptive closure.

> **[Definition 2 (Descriptive Closure)]**
>
> A descriptive system S is said to be closed with respect to an object when S can exhaustively reconstruct the structure the object possesses using only the elements of S and their relations, without introducing any new independent descriptive axis irreducible to S. When the reconstruction demands at least one independent descriptive axis irreducible to S, S is said not to be closed—to be non-closed—with respect to that object.

This "descriptive closure" is a structural analogy modeled on algebraic closure in mathematics. At the first tier, the field of real numbers is not algebraically closed with respect to square roots of negatives; it first closes in the minimal extension obtained by adjoining the imaginary unit i—the field of complex numbers as a quadratic extension. Definition 2 transfers this circumstance to description in general. Non-closure is not a defect of a descriptive system but an index that the structure of the object exceeds the expressive power of the system in question. Alongside it, we lay down the following norm concerning descriptive axes in general.

> **[Norm 1 (Load-Bearing Requirement on Axes)]**
>
> Each descriptive axis belonging to a descriptive system has standing within the description of an object only by being necessary for the reconstruction of some structure the object possesses. An axis demanded by no structure of the object does not belong to the description of that object.

Norm 1 bears this name because each axis belonging to a descriptive system must carry a load of its own within the description of the object—an axis that carries nothing has no place in the system. The demand runs parallel to the principle of parsimony in theory construction generally, epitomized by Ockham's razor—entities are not to be multiplied beyond necessity (for a contemporary treatment, Sober, 2015). The ground of Norm 1, however, lies not in an appeal to tradition but in the structure of the descriptive enterprise itself. Variation along an axis demanded by no structure of the object corresponds to no difference in the object. Such an axis describes nothing about the object at hand and is nothing but the descriptive machinery running idle—it fails to belong to the description not by a virtue of economy but by lacking descriptive content. Norm 1 does not, be it noted, prohibit the introduction of rich structure in general. If the structure of the object demands it, just so many axes are added as are demanded. What Norm 1 excludes is undemanded structure alone.

With these two in place, the policy this paper adopts is made explicit as the following principle.

[Principle 1 (Principle of Minimal Extension)]

When a descriptive system S is not closed with respect to an object P, the adequate description of P (i) unavoidably demands the introduction of independent descriptive axes irreducible to S, and (ii) that introduction is confined to the minimal number of axes necessary to resolve the non-closure.

Principle 1 is not an axiom adopted at will; it follows from Definition 2 and Norm 1. Clause (i) is an implication of Definition 2—non-closure is defined precisely as the demand for independent descriptive axes. The minimality of clause (ii) follows from Norm 1—axes introduced beyond the number necessary to resolve the non-closure are demanded by no structure of the object and, by Norm 1, do not belong to the description of that object. Hence the independent axes added to an adequate description remain exactly as many as the non-closure demands. The standing of Principle 1 is that of a consequence of these two premises, and it contains no claim about the reality of the axes introduced. The quadratic extension from the real to the complex field, be it noted, is not the ground of this principle but its most perspicuous instance—the complex field is nothing other than the minimal extension field that algebraically closes the reals. This order—the principle is not supported by analogy from mathematics; the mathematical case is one application of the principle—is preserved intact when the principle is applied to the structure of time.

The three-tier structure can be read as the recurrence of this principle. At the first tier, the field of reals is not closed with respect to the structure of the complex numbers, and adding one independent axis—the imaginary axis—yields the complex plane. At the second tier, the real dimension D appearing in description is not closed with respect to the totality of the object's existence, and adding one independent axis—the imaginary dimension iD—yields Z = D + iD (Muranushi, 2026a). At the third tier—the subject of this paper—the real time axis t is not closed with respect to the coexistence of phases in the present (as §05 will argue), and adding one independent axis—the imaginary time axis it—yields T = t + it. At each tier the independent axis added is always one, and this fixes the two-dimensionality of each structure not arbitrarily but as a consequence of the Principle of Minimal Extension.

This tiered structure, and the Principle of Minimal Extension that generates it, are the premises of the argument of this paper. What follows addresses how non-closure obtains at the third tier.

03 Lineages of the Concept of Imaginary Time and the Position of This Paper

The term imaginary time carries distinct traditions in physics and in philosophy. To demarcate the thematic position of this paper, the principal lineages are surveyed.

3.1 Imaginary Time in Physics — Introduction as Technique

Imaginary time in physics is in every case introduced technically as an analytic continuation of real time. First, the Wick rotation: the procedure that turns the real time axis into an imaginary time axis by the substitution $t \to -i\tau$, carrying Lorentzian spacetime into Euclidean spacetime. It is established as an indispensable technique in finite-temperature field theory, in the convergence of path integrals, and in the computation of non-perturbative effects.

Second, the no-boundary proposal put forward by Hartle and Hawking (Hartle & Hawking, 1983). According to general relativity, the origin of the universe should harbor a singularity at which density and gravity become infinite, as shown by the Penrose–Hawking singularity theorems (Penrose, 1965). The no-boundary proposal holds that in the very earliest universe the time axis takes on an imaginary-time character, whereby the singularity may be avoided and a smooth, boundary-free origin given (for a popular exposition, Hawking, 1988). Here imaginary time is introduced as a technique for evading a mathematical breakdown—the singularity.

In these undertakings imaginary time is positioned as a computational technique, and its ontological status is carefully held in reserve. Physics shows that imaginary time lies at the structural core of the theory, but does not directly answer the question in what sense it is real.

3.2 Time in Philosophy — Objections to Linearity

On the philosophical side, objections to the received notion of linearly flowing time have been voiced across several traditions. Bergson, in the *Essai sur les données immédiates de la conscience* (Bergson, 1889), distinguished quantifiable, spatialized time from qualitatively experienced pure duration (durée), and showed the latter irreducible to the former. The 1922 Paris debate between Bergson and Einstein was the emblematic collision of the philosophical and the physical conception of time. McTaggart (1908) divided time into the A-series (past, present, future) and the B-series (earlier, later), and by casting doubt on the coherence of the A-series unsettled the self-evidence of the notion of linearly advancing time. In the phenomenological tradition, Husserl treated the threefold structure of time-consciousness (Husserl, 1928) and Heidegger ecstatic temporality (Heidegger, 1927), each describing structures that exceed the view of time as a punctual series.

3.3 The Formal Tradition of Two-Dimensional Time

The form itself of taking time as two-dimensional has an independent tradition on the side of analytic philosophy. Dobbs, in a two-part study (Dobbs, 1951), proposed formulating the extensive aspect of time as two-dimensional precisely in order to explain the phenomenon of the specious present. There, in addition to the usual earlier–later relation, a second temporal order structurally independent of it was posited, and the relation of the two was likened to the mutual orthogonality of coordinate axes. Broad, too, later acknowledged (Broad, 1959) that the specious present he had once discussed becomes clearer in the vocabulary of two-dimensional time. These early attempts owed part of their motivation to psychical research on precognition, and sought to grant the second temporal dimension a real and explanatory status. The conception of multi-dimensional time drew critical scrutiny in its own day (Bunge, 1958), and its lineage has recently been given historical reconstruction (Moravec, 2026).

3.4 The Position of This Paper

The imaginary time it of this paper occupies a position distinct from all the lineages above. Where physical imaginary time is introduced technically as an analytic continuation of real time, the it of this paper is demanded as a descriptive axis independent of t. Where the phenomenological tradition set out from a critique of the linear view of time, this paper inherits that critique while formally articulating its terminus as a two-dimensional structure of time. And where the tradition surveyed in 3.3 introduced another temporal series possessing its own earlier–later order, intending its reality, the it of this paper is not a second temporal order but a descriptive axis on which the coexistence of phases is placed; its relation is not a metrical or an ordinal structure, but remains the structural orthogonality—the mutual irreducibility—of Definition 3 (§04).

Among contemporary discussions that thematize the coexistence of phases head-on stands Fine's fragmentalism (Fine, 2005). Where Fine embraces the coexistence as a fragmentation of reality itself—mutually incoherent fragments obtaining without integration into a single whole—this paper treats the coexistence as a circumstance on the side of description. The two diverge as the ontological and the descriptive response to one and the same phenomenon.

Table 1 summarizes the contrasts. Each lineage has grasped thickness or coexistence in a vocabulary of its own. But the point that placing that thickness demands an axis orthogonal to t—and demands it, moreover, as a non-real descriptive axis—is thematized in none of these lineages, and this one point is the subject of this paper.

[Table 1 Lineages Concerning Thickness and Coexistence, and the Position of This Paper]

Lineage	Grasp of thickness / coexistence	Status of the orthogonal axis
Phenomenology (Husserl, 1928)	Described as the threefold structure of retention and protention	Not thematized
Specious-present debate (James, 1890; Dainton, 2000)	Thematized as the width of the present	At issue only whether it reduces to a t-interval
Hypertime (Dobbs, 1951; Broad, 1959)	Formulated as a second temporal order	Introduced as a real dimension
Fragmentalism (Fine, 2005)	Embraced as fragmentation of reality	A structure on the side of reality
This paper	Coexistence of phases irreducible to t	A non-real descriptive axis demanded by non-closure

04 The Core Formula T = t + it

The foundation of this paper condenses into a single formula.

[Definition 3 (Complex Time)]

Complex time T is defined as the composition of a real-time-axis component t and an imaginary-time-axis component it; that is, $T = t + it$. Here t and it constitute independent descriptive axes, neither reducible to the other. This paper calls the relation structural orthogonality, after the relation of the real and imaginary axes on the complex plane. It does not mean geometrical orthogonality in an inner-product space; it is a formal specification expressing the mutual irreducibility of the two descriptive axes.

This formula transfers to the structure of time the relation between the real dimension D and the imaginary dimension iD in the second tier's $Z = D + iD$. Just as, on the complex plane, the real and imaginary axes are orthogonal and a complex number is given as a point on that plane, so the t axis and the it axis are orthogonal, and complex time T is given as a structure on that temporal plane.

A point of notation must be made explicit. The it in the formula $T = t + it$ is neither a quantity obtained by multiplying the real time value t by i, nor a quantity derived from t by analytic continuation in the manner of a Wick rotation. The same letter t is used as a notational convention displaying the structural correspondence with the series formula $Z = D + iD$; just as the real and imaginary coordinates of a complex number $z = a + bi$ take their values independently, so the t-axis and it-axis components are independent. Accordingly, the reading that takes the two as a line generated by a single quantity t—the reading that collapses $T = t + it$ into the one-dimensional ray $t(1 + i)$—is not contained in the formula of this paper.

What the complex form carries here, moreover, is the placing of this independence under a coordinate form of the same type as the series—not the derivation of any claim from the form itself. The argument for the independence is borne by the irreducibility argument of §05, and even with the notation $T = t + it$ removed, the claim that the description by t alone does not close stands on its own. Indeed, all this paper employs is the two-dimensional coordinate structure spanned by two independent axes; the algebraic properties proper to the imaginary unit i—$i^2 = -1$ and the rotation it fixes on the complex plane—are used in none of the paper's arguments. The epithet "complex" is not a claim attributing the algebraic structure of the complex field to time, but a notational adoption preserving the structural correspondence with the series. Whether the algebraic properties of i could do work of their own in the structure of time—whether, for instance, transitions between phases could be formulated as rotations—remains an open question outside the scope of this paper. Nor does this notational adoption conflict with Norm 1: what Norm 1 governs is the introduction of descriptive axes attributed to the object, not the choice of the symbolism in which introduced axes are written down. The only structure this paper attributes to the object is the independence of the two axes, and this, as §05 shows, is demanded.

[Proposition 1]

Observed temporal phenomena are described as the composite behavior of complex time T. A description of time that closes with the real time axis t alone is an incomplete description confined to the projection of T onto its real-time side.

The time we experience is always T. The stance that isolates t and handles it alone is positioned as a reduction that identifies time with t, the projection of T onto its real-time side. t and it do not stand opposed; phenomenal time appears as their composition T.

This projective relation is a descriptive relation. Describing the structural whole T from the side of real time yields t; describing it in its structural wholeness yields T—what is at issue is not a ranking of t and it but a difference of descriptive strata. Hence t is not a false time but a legitimate aspect of T, and it is not its opposite but its complement. This corresponds to the way the real dimension D observed at the second tier appears as the real-side projection of Z = D + iD. This projective view, be it noted, touches the four-dimensionalist idea in the analytic philosophy of time, which takes successive events as cross-sections of a four-dimensional whole; but what this paper places as the base of the projection is the it axis orthogonal to t, not the totality of events arrayed along t.

This paper does not assert that imaginary time it is real. What it asserts is the descriptive circumstance that the description of real time t does not close on its own (in the sense of Definition 2), and that an adequate description of time demands an independent descriptive axis it orthogonal to t. In this respect the paper carries into the structure of time the stance of this series (Muranushi, 2026a), which argues that the description of the real dimension does not close by itself.

05 The Proper Domain Described by Imaginary Time it

What imaginary time it describes is the circumstance that different phases coexist within a single present. This section argues that this coexistence cannot be structurally described by the t axis alone—the irreducibility set up as Proposition 2.

it describes the "layer" over a point on the t axis. The experience of the present is not exhausted by the phase now occurring. There the phase just passed still overlaps, and the phase to come already bites in. Onto the phase now occurring, these phases are superimposed within one present, and the present is not a widthless point on the t axis but a layered structure in which several phases coexist in the direction of it. This coexistence of phases cannot be reduced to before-and-after relations on the t axis.

That the present has width is a structure discussed in analytic philosophy as the specious present (James, 1890; Dainton, 2000). The contribution of this paper, however, does not lie in asserting the existence of thickness but in the point that the thickness forms an orthogonal structure irreducible to t; no re-adjudication of the specious-present debate is intended. The recognition that the present possesses thickness is not proprietary to phenomenology; the analytic literature on the specious present has grasped the same circumstance in a vocabulary of its own. What this paper newly asserts is the structural point that placing that thickness demands an independent descriptive axis orthogonal to t.

Decisive here is that this coexistence of phases is not recovered by raising the resolution of t. The paper sets this up as the following proposition and argues for it below.

> **[Proposition 2 (Irreducibility of Phase Coexistence)]**
>
> The coexistence of phases in the present is not reconstructed by the internal resources of the real time axis t alone—neither by any division into instants nor by any replacement with an interval.

This coexistence of phases appears in experience as the layered thickness of the present. In what follows, the word thickness is used only when the same circumstance is designated from the side of its experiential guise—the principal concept remains the coexistence of phases, of which thickness is the appearance. The argument itself is not a formal mathematical proof but a structural claim at the level of phenomenological description. What is in question is not a property of a mathematical object, the set of instants, but the describability of the structure possessed by the lived present. The argument is in two steps.

First, what a division of t yields is only a denser sequence of widthless instants separated by shorter intervals. No instant, taken by itself, has thickness—to lack width is the very definition of an instant. However densely widthless elements are stacked, no thickness arises from them.

Second, what the t axis represents is the before-and-after relation among instants—a relation between two positions. What constitutes thickness, by contrast, is the relation between one present and the phase given, within that present, as passing away. The latter is not a relation between two t positions but a relation, interior to one present, between the present and the non-present. The two differ in category. However fine the before-and-after relations are made, they do not convert into the interior structure of the present.

Against this, an objection suggests itself: the retention of phases passed could be constructed from t alone, as the quantity obtained by integrating past values under a decaying weight,

$$R(t) = \int_{-\infty}^{t} w(t-s)\, x(s)\, ds$$

where R(t) sums the experience at each past moment up to the present, weighting the older the smaller, and expresses the past that remains in the present as a single quantity. This construction is the strongest form of the reduction, but it falls into a dilemma. If the integral is evaluated offline over the whole signal, no privileged "now" exists there, and what is described is not the lived present but an objectified trace—the first-person standpoint to which the thickness belongs, the construction has surrendered in advance. If, conversely, a particular present t is read off as the upper limit, then that "now"—an indexical standpoint that is none of the past positions s—has been reintroduced, and it is the very orthogonal structure that was to be eliminated. The reduction succeeds only if the thickness can be carried over into t intact. But the first horn drops the very thing to be described—the lived present—and the second horn, in order to keep it, calls the orthogonal standpoint back. On either horn, the thickness is not reduced to t.

This upshot admits a more compact statement. What the integral R(t) returns is not the thickness itself but a value that folds the thickness into a single scalar on the real time axis—the projection of the thickness onto the t side. What a construction from t alone in principle drops is precisely the dimension lost in this projection. The model of the reduction exemplifies, by its own structure, the view that t is a projection of T (Proposition 1).

What is at issue here, be it noted, is not the subjectivity or the psychology of an observer, but the point that the indexical present itself remains an ineliminable structural presupposition in the construction of retention. That the indexical "now" is not exhausted by non-indexical descriptions such as dates and positions is an instance, within the structure of time, of the debate over the essential indexical (Perry, 1979). The dilemma of this paper is nothing other than that irreducibility of the indexical surfacing on the time axis.

The reduction has one further form besides the convolution of retentions: the position that explains the thickness not by construction out of widthless instants but by the fact that experience itself extends over a short interval of the real time axis—extensionalism (Dainton, 2000). Since experience is held to have width from the outset, the first point above, which dismissed the densification of instants, does not touch this position. But the difficulty recurs in altered form. What an interval provides on the t axis is the before-and-after relations of the phases within it. What constitutes the thickness, by contrast, is the relation of those phases co-belonging to one present, and this co-belonging differs in category from before-and-after relations interior to the interval. If co-belonging were exhausted by "falling within the same interval," any interval on the t axis would constitute a present; but no principle fixing which interval constitutes one present exists within the metric and the order of t. Indeed, the relation of diachronic co-consciousness, to which extensionalism entrusts this binding, is introduced precisely as a relation not reducible to before-and-after. Extensionalism thus already presupposes a binding relation exceeding the order structure of t, and the explanation that replaces the thickness by an interval leaves unexplained the very structure that makes an interval one present.

Thus the two reductions—the convolution of retentions and the extension over an interval—run up against residues of the same type: the former leaves an ineliminable indexical standpoint, the latter a co-belonging irreducible to before-and-after. But one further step is needed here. Why is what this residue demands an axis—a descriptive dimension with internal structure—rather than a mere marker of standpoint? The strongest position holding a marker to suffice explains the indexical "now" by folding it back onto the very occurrence in which the word is uttered (Mellor, 1998). On this account, an utterance of "It is raining now" is true if and only if it is raining at the time the utterance is made—"now" refers to nothing beyond the very time at which each utterance occurs, and an independent dimension appears unnecessary. But what this account provides are the conditions under which sentences containing "now" come out true—not the layered structure of the present. The occurrence of an utterance is a point on the t axis and has, of itself, no thickness—the identification with a point drops, once again, the very layer that was to be placed. The "now" that the two reductions leave behind is not a widthless point. It is an internally articulated layer in which the phase passing away and the phase to come overlap, each at a position of its own—as more deeply sunken, as more shallowly encroaching. What can place something that possesses a continuous system of positions is not a marker but an axis. By an axis is meant here the minimal structure that can carry a one-parameter system of positions—a single independent descriptive dimension. That the structure demanded is neither poorer nor richer than this is fixed as follows. A mere marker, as just seen, cannot carry the continuous system. Richer structures such as manifolds and bundles, on the other hand, import structure that the coexistence of phases does not demand, and thereby conflict with Norm 1 (the load-bearing requirement on axes)—for the overlap of phases is articulated by a single parameter, more deeply sunken / more shallowly encroaching, and one dimension suffices for its placement. What the residue demands, then, is more than the marking of a point and less than a manifold—a single descriptive dimension on which the phases can take their positions: the it axis.

Hence the thickness of the present is structurally placed only on an independent axis it orthogonal to t. The operation that would close the description by t over the thickness presupposes, on the contrary, the very structure that cannot be closed.

With this, the non-closure of the description by t alone is confirmed, and, following Principle 1 (the Principle of Minimal Extension), exactly one independent descriptive dimension irreducible to t is added. This independence is nothing other than the relation called structural orthogonality in Definition 3, and the two-dimensionality of the structure of time—no more and no less—is given as a consequence of the Principle of Minimal Extension.

One point, finally, by way of anticipation. What this paper has described as the thickness of the present is, more generally, one special case of structural simultaneity—the coexistence, in the direction of it, of phases separated on the t axis. The coexistence of phases in the present is structural simultaneity in the case where the separation is very small. The description of the general circumstance, including cases of large separation, is the subject of a separate paper.

06 The Orthogonal Relation to Hartle–Hawking

This section makes explicit the relation of this paper's imaginary time it to imaginary time in physics through the nearest case, the Hartle–Hawking no-boundary proposal. The two use the same words, "imaginary time," yet their structural positions differ fundamentally.

[Table 2 Structural Differences between the Two Concepts of Imaginary Time]

Aspect	**Hartle–Hawking imaginary time**	**Imaginary time it of this paper**
Scope	Cosmological	Phenomenological–ontological
Mode of introduction	Wick rotation ($t \to -i\tau$)	Shift of descriptive stratum
Purpose	Mathematical avoidance of the singularity	Completion of the incompleteness of linear time description
Character	Computational technique	Structural extension of time description

Imaginary time in the Hartle–Hawking no-boundary proposal is a computational technique introduced as an analytic continuation of real time; its purpose is the avoidance of a mathematical breakdown, the singularity. The imaginary time it of this paper, by contrast, is demanded as a descriptive axis independent of t; its purpose is the completion of the incompleteness of linear time description. The two do not compete.

Rather, the very fact that physical imaginary time functions effectively as a technique is, from the standpoint of this paper, suggestive. The paper stands in the position of opening the question whether behind that effectiveness there may lie the two-dimensionality of the structure of time itself. This paper's imaginary time is therefore not an alternative theory to physical imaginary time, but a formal theory of time description that obtains independently of the adoption or rejection of any particular physical theory.

07 Conclusion

Against the received view that takes time to be a one-dimensional linear flow, this paper has proposed that time be described as a two-dimensional structure constituted as the composition T = t + it of the real time axis t and the imaginary time axis it. The ground condenses into one point: the circumstance that different phases coexist in the present is recovered neither by dividing the t axis nor by replacing it with intervals, and is not reduced to the before-and-after relations of t. The paper has argued for this irreducibility of phase coexistence through the difficulties respectively facing the two attempts to reconstruct it on the real time axis—the convolution of retentions and the extension over an interval (§05). The paper stops at exhibiting descriptive non-closure; the examination of what this implies concerning reality is entrusted to a separate paper occupying the position of a response within this series.

This paper is not a paraphrase, in another vocabulary, of the "temporal structure that linear time cannot capture" described by phenomenology. Nor is the form itself of formulating time as two-dimensional without precedent: it has prior attempts aimed at explaining the specious present (Dobbs, 1951; Broad, 1959). The claim of this paper lies not in the novelty of the two-dimensional form, but in the point that its axis is introduced neither as a second temporal order nor as a real dimension, but as a non-real descriptive dimension demanded by the non-closure of the description by t alone. To have shown that the coexistence of phases forms an independent descriptive dimension irreducible to t, and to have placed this under the form of complex time—herein lies the contribution of this paper.

As the extension of the number system from one dimension to two carried broad implications for mathematics and physics, so the extension of the understanding of time from one dimension to two demands a descriptive re-arrangement of the categories that stand on the premise of linear time. In particular, the general theory of structural simultaneity, which treats the circumstance of phases separated on the t axis coexisting, contains the phase coexistence of the present discussed in this paper as the special case of very small separation. Its formal refinement, and its structural integration with the concept of arising (okori) of the second paper, are the tasks that follow.

Viewed more broadly, the argument of this paper is positioned as one application—to the structure of time—of a single descriptive principle: a description that does not close demands, minimally, the independent axes that complement it (Principle 1). The same principle is already at work in the complex numbers of the first tier and in the Z = D + iD of the second; imaginary time is the third case in that series and, at the same time, the place where the Principle of Minimal Extension is first made explicit as a principle. From this vantage, the paper is at once a study of time and the presentation of one instance of a more general framework concerning descriptive non-closure and its minimal extension. The full development of that generalization is a subsequent task of this series.

What this paper has shown is not the nature of time itself. It is a structure of the description of time: that the circumstance of different phases coexisting in the present does not fit within a one-dimensional description of time, and is placed only on a descriptive axis orthogonal to it.

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