

R-ealism at Its Limit

— Beyond Reductive Physicalism —

原題：実数主義の極北——還元的物理主義の果てに——

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Abstract

This paper defines as R-ealism the tacit ontology on which the world is exhaustively describable, and argues for the structural limit this position meets when it carries its own inner logic to the end. R-ealism equates being real with being formally describable—with R-describability as its representative form—and has run through both the natural science and the metaphysics of the modern period. But this dominance comes at a price. However precisely we describe an object, the description always leaves a surplus that has not yet risen into meaning; a description can record what has already taken shape, but can never contain the very arising by which that shape comes to be. R-ealism has banished this surplus outside the real as “something subjective” or as “an appearance to be explained away in due course.” Pursued to its limit, R-ealism runs up against the non-closure of description: in three regions—experience, meaning, and event—description necessarily fails to capture the arising that precedes its result. As a framework that positions this surplus without negating R-ealism, the paper sketches Extended Imaginary Number Theory ($Z = D + iD$; Muranushi 2026a) and proposes Extended Realism, which restores that surplus as the imaginary dimension iD on the side of the real. The conclusion: R-ealism is not false, but narrow.

Keywords: R-ealism; the non-closure of description; Extended Imaginary Number Theory; Extended Realism; reductive physicalism; structural realism; passive synthesis; contingency.

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01 Introduction: Defining R-ealism

1.1 The Place of This Paper: Its Relation to the Extended Imaginary Number Theory Series

This paper is positioned as a response, from the outside, to the Extended Imaginary Number Theory series (Muranushi 2026a, 2026b, 2026c). The first paper, “Extended Imaginary Number Theory: $Z = D + iD$ ” (Muranushi 2026a), advanced an ontology that describes the total being $Z(X)$ of an object X as a superposition of a real-dimensional component $D(X)$ and an imaginary-dimensional component $iD(X)$. Rather than defending the validity of that framework from within, the present paper takes as its object, from outside the series, the theoretical counter-position the series has tacitly confronted throughout—namely, the position holding that the world is closed within the real dimension D alone—and analyzes its internal logic.

The earlier papers in the series all followed a principle of structural avoidance—a policy of never speaking of the imaginary side directly and confining themselves to description on the real side—and so refrained from addressing the counter-position head-on. The present paper temporarily lifts this self-restriction and makes the counter-position its theme. The critical argument of this paper (§§ 02–04), however, relies solely on the internal logic of the counter-position and does not draw on any argument from the imaginary side. That an outside opens up when the counter-position carries its own logic to the end—this is the theme of the critical part. § 05 sketches how that opened outside is systematically received by the framework the earlier papers built ($Z = D + iD$). Up to § 04 we use the term D (the real dimension) merely for convenience; the imaginary dimension iD is introduced head-on only in § 05.

1.2 The Problem

Since the modern period, one tacit answer to the question of what the world is has attained a dominant position in both natural science and ontology. That answer is: the world is the totality of the measurable. Mass, length, time, energy, probability, quantity of information—all are defined as mappings into the real numbers R . And to “exist” in the world is, ultimately, nothing other than to possess a structure that can be captured by such an R -mapping. This answer is deeply embedded in the modern organization of knowledge, both as the methodological presupposition of natural science and as the tacit horizon of metaphysics.

The price of this answer’s dominance is not small. The position tacitly assumes that the world is exhausted by “what has been described and grasped as meaning.” Only what has been observed, put into language, and recorded as number—only what has thereby entered the domain of meaning—is real. Yet whenever we describe an object, however precise the description, the object is always accompanied by a surplus that has not yet arisen into meaning, that cannot even be put into words. Description always leaves something behind. The question is whether what is left behind is a mere “fact not yet known,” or something of another kind that the activity of description is structurally unable to exhaust.

The concern of this paper is to reexamine this tacit assumption that “description can close the world.” Does the position that makes describability the condition of the real truly exhaust the world? And if it does not, is what is left behind a mere deficiency of cognition, or a structural surplus that, while being part of the real, remains in principle prior to description?

This state of affairs has been a recurring object of critique in twentieth-century philosophy. Husserl, in *The Crisis of the European Sciences and Transcendental Phenomenology* (1936/1954), traced its origin to Galileo’s “mathematization of nature”; Whitehead grasped it as the “bifurcation of nature” (1920); Heidegger, as the dominance of “the mathematical” (1962)—each criticizing the same state of affairs from a different angle. Despite their differences of expression, these three critiques jointly sketch the contour of a single metaphysical position. This paper names that position R-ealism. The genealogy is developed in detail in § 02.

1.3 The Reason for the Name

There are two reasons for the name. First, the core of this position lies in the requirement that the real content of the world be exhausted by physical quantities describable as mappings into the real numbers \mathbb{R} . Mass, charge, momentum, and temperature are all described as mappings into \mathbb{R} , and here “to be real” becomes synonymous, ultimately, with “to be mappable into \mathbb{R} .” That is, this position is in substance a real-number-dimensionalism, which this paper abbreviates as “R-ealism.” Second, the name is polemical, not neutral: R-ealists call themselves “realists” or “naturalists,” but as this paper shows in § 05, they are better situated as partial realists who admit, of the real, only the side that fits within description. This is not a premise at this stage but a conclusion the following argument aims to support.

1.4 Why “R-ealism” Rather Than “Physicalism”

This paper calls its object of analysis “R-ealism” rather than “physicalism” in order to preempt the objection that the argument attacks a straw man—“physicalists do not claim that the world consists of \mathbb{R} alone.” The two are defined at different levels. Physicalism is defined by its domain of objects (what exists in the world is what physics deals with), whereas R-ealism is defined by a formal constraint: it places the condition for a property’s being real in the formal property of mappability into \mathbb{R} .

This distinction is decisive because many contemporary forms of physicalism have expanded their domain of objects from “the objects of physics” to “mathematical structure in general.” However far the domain is expanded, so long as the formal condition of reality is placed in “being transcribable into a formal description,” it belongs to the target of this paper. What this paper asks is not “does the world consist of physical objects?” but “is being real exhausted by being formally describable?” (see § 3.1 for details).

1.5 The Scope of This Paper

Let us state the structure and scope of this paper. The structure is as follows. § 02 reconstructs the genealogy of R-ealism as a formation in the history of thought. § 03 formalizes the logical structure of R-ealism as a “completeness requirement” and a “closure of the

world,” and situates reductive physicalism as one of its implementations. § 04 analyzes the three regions in which R-ealism’s description fails to close, drawing on Husserl’s passive synthesis, Kripke’s problem of rule-following, and Meillassoux’s argument from contingency. § 05 sketches Extended Imaginary Number Theory as a framework that systematically situates the surplus left behind there. § 06 states the conclusion.

The responsibility of this paper is confined to demonstrating the non-closure of description itself—to showing, from the internal logic of the counter-position, that non-closure belongs to the structure of the real and not to a deficiency of cognition. It shows that writing the surplus left behind as iD is the most economical option that remains, but it does not enter into any positive determination of the internal structure or ontological status of iD. These are left to the earlier papers in the series (Muranushi 2026b, 2026c) and to separate work. This paper fully grants the methodological efficacy of R-ealism—the cumulative success of natural science since the modern period. What it rejects is solely R-ealism’s claim of metaphysical closure. R-ealism is effective as a method, but incomplete as an ontology.

02 The Genealogy of R-ealism: From the Mathematization of Nature to the Computational Worldview

R-ealism is not a position founded by any particular philosopher. Rather, it is a metaphysical sediment that has formed gradually over four centuries, in parallel with the methodological success of natural science since the modern period. This section sketches the history of this sedimentation in five phases.

2.1 The Mathematization of Nature in Galileo

The starting point of R-ealism can be located in Galileo Galilei’s famous proposition—that the great book of nature is written in the language of mathematics, its characters being triangles, circles, and other geometric figures (Galilei 1623). This proposition is no mere metaphor. For Galileo, to know nature was synonymous with grasping nature as mathematical structure. To describe the fall of a body as a function of time and distance, and to develop that function as an algebra of R-values—this is the legitimate form of natural knowledge.

Galileo’s step contains two implications. First, a tacit ontological claim: the ontological content of nature is exhausted by what is mathematically graspable. Second, a tacit epistemological claim: what cannot be grasped mathematically—color, scent, warmth, value—belongs not to objective nature but to the side of the subject. When these two claims are joined, the ontological program of modern science is established—what is real is what is R-describable.

2.2 Husserl’s Discovery of the Genealogy

In § 9 of *The Crisis of the European Sciences and Transcendental Phenomenology* (1936/1954), Husserl laid bare the ontological price of this Galilean program. According to Husserl, Galileo did not discover “nature itself” but reconstituted nature in a particular

form. This procedure of constitution covers over the lifeworld—the world as perceived, felt, and practiced—with its mathematized residue.

This procedure, which Husserl calls the “mathematization of intuition,” proceeds in two stages. In the first stage, the qualitative experience of the lifeworld—the vividness of a color, the resonance of a sound, the warmth of the skin—is replaced by a corresponding quantitative description—wavelength, frequency, temperature. In the second stage, this replacement is inverted: the quantitative description is repositioned as “the true real,” and qualitative experience as “subjective representation.” Thereby the mathematization that was originally a tool for ordering qualitative experience is stood on its head as the ontological foundation of qualitative experience itself. Husserl called this inversion the “crisis.” R-ealism is the completed form of this inversion.

2.3 Whitehead’s Critique of the “Bifurcation of Nature”

Whitehead grasped the same state of affairs as the “bifurcation of nature” (1920, ch. 2)—modern science divides the world into “the nature we are aware of” (the green trees and the song of birds) and “the nature that is the cause of awareness” (the motion of molecules and the wavelength of light). For Whitehead these are not two separated natures but merely two “prehensions” within one nature. The error of R-ealism lies in privileging only the R-describable side as “true nature.”

2.4 “The Mathematical” in Heidegger

Heidegger (1962) defined the essence of modern science as the dominance of “the mathematical”—an ontological stance that, prior to any particular mathematical proposition, grasps objects within a framework decided in advance. Natural science did not discover nature itself but organized nature in advance as “that which can be put to mathematical questioning.” What appears as nature is decided not by nature itself but by the stance of the one who poses the question. R-ealism is the position that equates this “that which can be put to mathematical questioning” with “all that is.”

2.5 From the Twentieth Century On: From Physicalism to the Computational Worldview

In the latter half of the twentieth century, the Galilean–Newtonian program came to have two metaphysically explicit successors. The first issues from Quine’s naturalism (1969) and was refined by Smart (1959), Armstrong (1968), Lewis (1966), and Kim (1998): physicalism. Reductive physicalism in particular is the most logically coherent contemporary form of R-ealism, and is treated in § 3.6. The second successor is the computational worldview developed from the end of the twentieth century into the twenty-first: Wolfram’s “new kind of science” (2002), Tegmark’s Mathematical Universe Hypothesis (2014), and Lloyd’s thesis that the universe is a quantum computer (2006)—all attempts to grasp the world as a computable structure. The description-systems they employ are not confined to real values but include complex numbers and discrete structures. What matters is not whether the target of the mapping is the real numbers, but that the world is required to be exhausted within some formal description. The relation between this generalized stance and R-ealism is reorganized in § 03.

Furthermore, in the twenty-first century R-ealism has extended beyond the frame of philosophical theory into the social and practical dimension. Evidence-based policy - making, the numerical quantification of moral value via QALYs in effective altruism (Singer 1972; MacAskill 2015), optimization decisions by AI, the evaluation of human activity by KPIs—all are implemented forms of R-ealism. The mathematization of nature that began with Galileo has now reached the mathematization of the human being itself. That is, R-ealism unfolds across multiple forms with different domains of objects—mind (physicalism), cosmos (computational metaphysics), information (information ontology), ethics (evidence-based ethics)—while sharing one and the same metaphysical structure. These five phases (§ 2.1–2.5) display the historical thickness of R-ealism from Galileo to today. It is not a mere philosophical position but a metaphysical sediment deeply embedded in the entire organization of knowledge since the modern period.¹ The next section analyzes the logical structure of this sediment.

03 The Logical Structure of R-ealism: Closure into R and the Completeness Requirement

This section makes explicit, as logical structure, the historical formation sketched in the previous section. Formally, R-ealism begins from the following definition.

Definition 1 · R-ealism

Every real constituent of the world W is completely described by the set P_R of physical quantities describable as mappings into the real numbers R . That is, to be real is, ultimately, to be R -describable.

R-ealism is the most naive and pure conception of reality presupposed by modern physical science—what is real is exhausted by what can be measured and recorded as number. Mass, charge, temperature, and probability are all described as mappings into R . This paper sets this position as the target of its analysis because it is the most naked form of the broader stance that “closes the real within formal description.”

3.1 Scope: Relation to Structural Realism and Information Ontology

Here a qualification is needed. Sophisticated contemporary thinkers do not say that the real is closed within the real numbers themselves. Tegmark (2014) takes the world to be mathematical structure itself; Ladyman & Ross (2007) place relational structure, and Floridi (2011) informational structure, at the foundation of the real. Saying “we are not speaking of mere quantities,” they dodge naive R-ealism. Yet these positions too share one point—they equate being real with being transcribable into some formal description-system. Mathematical structure, relational structure, and informational structure are all, equally, description-systems that can be operated upon by rule. The argument of

1. It should be noted that the R-ealism this paper defines formally has a broader cultural correlate—an attitude that shuts the invisible and the immeasurable out of the real from the start. Naive materialism and popular atheism may be seen as instances of it, but these belong to a different axis, that of faith and worldview, and hold independently of the claim of formal-descriptive closure. The scope of this paper is confined to the latter, formal claim; consideration of the former, cultural and attitudinal dimension is left to separate work.

this paper therefore reaches all of these, whether the target of the mapping is real numbers, mathematical structure, or information. For the non-closure of description discussed below (§ 04) derives not from the choice of description-system but from the very stance of closing the real within description. R-ealism is the simplest representative of that stance, and this paper treats it as the representative. In what follows, for clarity, we will where needed call this general stance formal-descriptive closure.

For the same reason, the natural objection of a reader versed in physics—“physics has long used complex numbers; since the wavefunction of quantum mechanics takes complex values, the real is not closed within R at all, so R-ealism misses its mark from the start”—also mistakes the target of this paper. The issue is not the choice, within a description-system, of whether to use real or complex numbers, but the stance of closing the real within some formal description. The complex numbers of physics are tools internal to a description-system, ultimately connecting to describable quantities such as observables and probabilities. By contrast, the imaginary dimension this paper calls iD never appears within any formal description—it is the condition for description to obtain, and never comes forth on the side of the described content. The two share the sign of the imaginary only as a metaphor; their ontological status is exactly opposite (this metaphor is taken up again in § 5.1).

3.2 The Two Constituents

In what follows we analyze the logical structure of this position in terms of R-ealism. As stated above, the conclusion does not depend on the choice of description-system and reaches every position that closes the real within formal description. The R-ealism of Definition 1 contains two constituents. The first is that any real content of the world possesses R-describability (hereafter the “R-mappability thesis”). The second is that the set of R-describable things exhausts the real content of the world (hereafter the “completeness thesis”). The R-mappability thesis is, in itself, a relatively weak claim. Many non-physicalists accept it. For example, dualists who admit that mental events depend on brain states (parallelism or epiphenomenalism), as well as emergentists, can be compatible with it. What characterizes R-ealism is the second, the completeness thesis.

3.3 The Completeness Requirement and Hempel’s Dilemma

The completeness thesis is the most fiercely contested point of contemporary physicalism. Hempel’s dilemma (1969) brings it to light—if “physical” in physicalism refers to what current physics talks about, then since physics is still in progress, physicalism may be empirically false. If it refers to a completed future physics, then since the content of that physics is unknown, the content of the claim is indeterminate. Many contemporary physicalists choose here a somewhat vague third way (a posteriori physicalism): “physical” refers to the properties described by current and future physics. R-ealism, however, chooses a stronger response—it defines “physical” as R-mappability itself, independent of both current and future physics. By this formal, ontological determination, R-ealism evades Hempel’s dilemma and at the same time narrows its own empirical refutability. A refutation of R-ealism can no longer be constructed from any particular physical pro-

position; it is possible only by exhibiting the limit of R-describability itself. As § 04 shows, this determines the structure of R-ealism's self-undoing at its limit.

3.4 The Three-Layered Structure of Closure

The completeness thesis implies three layers of closure.

(i) Causal closure—every event in the physical world has an R-describable cause sufficient to fully explain it (Kim 1998). This layer is strongly supported by the conservation of energy and momentum in modern physics. This paper does not contest this layer.

(ii) Explanatory closure—every event is, in principle, explicable as a combination of R-describable properties. At this layer, the epistemological concept of explanation is made internal to the language of R-description. To explain an event e is to present the R-describable mechanism that produces e .

(iii) Ontological closure—what exists in the world are only the bearers of R-describable properties. Properties such as color, pain, beauty, norm, and meaning are each nothing but combinations of R-properties (nothing over and above).

These three layers increase in the strength of their requirement as one proceeds (i) \rightarrow (ii) \rightarrow (iii). The first is an empirical proposition, the second a methodological proposition, the third an ontological proposition. What this paper takes as its problem is how the transition from the second to the third layer—the leap from explanatory closure to ontological closure—is justified.

3.5 The Logic of the Leap

R-ealism justifies this leap by combining the supervenience thesis with reductive identification. The structure of the argument is as follows.

- (1) Any fact f supervenes on an R-describable fact p .
- (2) Since it supervenes, f is determined by a combination of p .
- (3) Yet if what determines and what is determined are distinguished, what is determined may have an independent ontological position.
- (4) To prevent this, the relation between f and the combination of p must be understood not as a mere relation of determination but as a relation of identity.
- (5) Therefore f is identical with the combination of p (type-type reduction).

Step (4)—the move from supervenience to identity—is not logically necessary. No logical contradiction arises if a supervening fact is not identical with the physical base on which it supervenes. Indeed, non-reductive physicalists construct their position precisely by refusing (4). R-ealism adopts (4) not from logical necessity but from the metaphysical goal of completeness of world-description. Yet this leap summons a structural limit into the interior of R-ealism. As § 04 shows, however much description is refined, description always leaves behind the “layer prior to the arising” of what it describes—the non-closure of description.

3.6 The Contemporary Form of R-ealism: Reductive Physicalism as One Implementation

The logical structure extracted up to § 3.5 attained its sharpest form in the philosophy of mind of the latter twentieth century. The identity theory and functional reduction of Smart (1959), Place (1956), Armstrong (1968), and Lewis (1966), and above all Kim's reductive physicalism (1998, 2005), derive from the supervenience thesis and the causal-closure thesis a dilemma of mental causation—either mental events overlap with physical causes, or else they are causally inert—and reduce mental events to type-type identity with R-describable brain events. This is the clearest implementation of R-ealism in the domain of mind. But Kim-style reduction is not its only contemporary form. As seen in § 2.5, the same three-layered closure structure is implemented across multiple forms with different domains—mind, cosmos, information, ethics. Hence the non-closure of description in the next section is not a problem peculiar to Kim-style reduction; it inheres in the very stance of closing the real within description and appears across all these domains.

04 The Internal Limit of R-ealism: The Non-Closure of Description

When R-ealism carries its logic to the limit, it runs up against one structural fact that it cannot answer within its own frame. It is this: however much description is refined, the object is always accompanied by a surplus that has not yet arisen into meaning. This section calls this the “non-closure of description,” states it first as a principle, then examines the three regions in which it appears, and finally fixes how this paper's surplus differs from prior residue accounts.

Principle (The Non-Closure of Description)

No attempt to describe an object constitutes, by itself, a closed description. What description can capture is confined to what has already arisen into meaning—what has been observed, put into language, and taken shape as a concept. But the object is always accompanied by a structural surplus that such meaning-formation does not reach, that cannot even be put into words. In terms of R-ealism: what R-description can capture is only what has already been articulated as quantity, and the surplus prior to articulation remains outside R-description.

Here we must preempt and dismiss the most natural objection from the side of R-ealism. The objection runs: that description is after the fact—that finishing the description of an event comes after the event's occurrence—is a trivial fact about the procedure of cognition. The “surplus” left behind is, in short, a “fact not yet described,” an epistemic deficiency that refinement of description will in principle fill, not a gap on the side of the real.

This objection, however, mistakes the status of the surplus. The point is not the procedural matter that description comes about belatedly in time, but the matter of what the activity of description is structurally unable to contain within itself. Suppose we try to recapture some arising itself within description. But the moment that attempt succeeds and something is written down, what is written down has become yet another “arisen

result.” And the prior side from which that result arises retreats one step further outward. The operation of trying to contain the arising within description produces, from where it has contained, a new prior side again, and never catches up. The surplus left behind is not a residue that shrinks as description is added, but a structural displacement that the operation of description itself reproduces at each iteration. We call this hereafter the regress argument.

Therefore the surplus is not a “fact not yet known”—what can be called “not yet known” has, at that point, already entered the domain of meaning as “something knowable” and belongs to the side of the in-principle-describable D. What this paper points to is the layer prior to being put into words in that way, the layer that description, at each turn, fails to catch by a step. Latent variables, tacit knowledge, and measurement error are all residues internal to the framework of description, not this surplus. The surplus is the name of the structure whereby the framework of description itself—however much it is refined—fails to arrive in time at its own arising.

This principle does not depend on the choice of system used for description. Replacing real numbers with complex numbers, relational structures, or information does not move the limit (§ 3.1). So long as the activity of description can capture only what has been articulated as meaning, the surplus prior to articulation is left behind whatever description-system one chooses. R-ealism, taking the simplest description-system—the real numbers—is the case in which this limit appears most nakedly.

This surplus accompanies every scene of the object, but there are three regions in particular where R-ealism’s description appears to break: the region where experience arises, the region where meaning arises, and the region where an event arises. We examine each in turn.

4.1 Prior to the Arising of Experience: The First Region

When we perceive something, the experience already takes a form that is articulated and can be put into language. “Red,” “painful,” “warm.” To avoid mistaking the target here, a distinction must be drawn. R-ealism does not admit the felt quality itself as real—it either reduces it to wavelength or neural state, or banishes it outside the real as “subjective representation” (§ 2.2). Hence the classic objection that “R-ealism leaves out qualia” (Jackson 1982) is, in itself, legitimate. But the target of this paper is not there. What this paper presents in § 05 as Extended Realism is a position that instead restores articulated quality to the side of the described result (D) as real, and then asks after a further, deeper layer at which description still does not close. What must be asked is not whether the articulated “red” can be described, but lies prior to that “red” arising. Whereas the qualia argument is a dispute over “the reality of described quality,” what this paper takes aim at is the layer prior to any quality’s coming to obtain as meaning; the two do not compete—the latter lies one step prior to the former.

What Husserl in his later work called “passive synthesis” (Husserl 1918–1926/2001) was the layer of synthesis that comes to obtain of itself on the horizon of consciousness, prior to active, thematic grasping. Before something is articulated as a “red object,” something is already being passively synthesized—this paper reads the layer prior to that synthesis

as a structure appearing nowhere in articulated description. Husserl himself, however, regarded this layer as describable through transcendental phenomenology. What this paper borrows is not his thesis of describability but the structure extractable from his analysis—the structure whereby, prior to articulated experience, there is a layer that prepares it yet does not itself appear in articulated description. What Merleau-Ponty depicted in the body schema and motor intentionality was likewise a pre-reflective structure preceding objective content. Perception has already responded before the subject judges.

R-ealism's description can capture the arisen result (articulated experience). But it cannot capture the arising itself—the layer prior to that, where meaning is just taking shape. For by the time the description obtains, the arising has already finished. The first region shows that this “prior side of the arising” structurally escapes description.

4.2 Prior to the Arising of Meaning: The Second Region

R-ealism tries to reduce meaning and rule to the description of facts. The meaning of a word is reduced to the regularity of use, a norm to the aggregation of preferences. But this reduction runs into a common difficulty. As Kripke showed by reconstructing Wittgenstein's problem of rule-following, one cannot uniquely extract, from a finite set of cases, a rule that applies to infinitely many (Kripke 1982). From any finite set of examples, several different rules are equally extractable. Which rule one is “following” is not fixed however many descriptions of cases one piles up. A rule is a normative constraint—“one ought to continue thus”—but the description of facts contains no “ought” whatever (as Hume argued that one cannot derive an ought from an is).

Rule and meaning, once they have arisen, belong to the describable side D. But the arising itself, by which a rule arises from finite examples (described facts), is not contained in the description of the examples. Description can always record an arisen meaning after the fact, but cannot contain within itself the very working by which meaning arises. The second region shows that the coming-to-obtain of meaning is not contained in the description of the very examples that bring it about—here the “prior” is not temporal precedence but a constitutional priorness: the description of the result cannot contain within itself the very working that makes it a result.

4.3 Prior to the Arising of the Event: The Third Region

R-ealism describes the world as law. The motion of a body follows an equation; a quantum state follows a probability distribution. These descriptions give what can happen (the space of possibility). But what description gives reaches only as far as the space of possibility—the totality of what can happen. That some one of them actually happens, that this here-and-now has arisen, cannot be derived from the description of possibility. However precisely one gives the set Ω of possible events and their probability P , why this event among Ω became actual is not written within the description (Ω, P) . As Meillassoux argued, this is not a deficiency of information but a structural state of affairs in which the principle of sufficient reason—that every event has a sufficient reason—cannot be grounded within the frame of description. (The invocation of Meillassoux here remains a structural borrowing of the account of contingency and does not commit to his anti-cor-

relationist position. What this paper borrows is solely the structural point that occurrence itself cannot be grounded in the description (Ω , P.)

Here too the configuration is the same. An event that has happened belongs, after it has happened, to D as a describable fact. But the occurring itself, by which an event happens—the transition by which a possibility arises into actuality—does not appear within the described space of possibility. Law can exhaust “what can happen” but cannot contain within itself “what actually happens.” The third region shows that occurrence precedes description.

4.4 The Structural Commonality of the Three Regions: An Independent Convergence

Experience, meaning, event—the three regions concern different scenes. Yet they share a common structure. In each, what description captures is the “arisen result,” and what description fails to capture is the “arising itself.”

Here we must confirm that the argument of this section is not a mere repetition of a definition. The claim that “the prior side of the arising cannot be described” might be suspected of circularity—did we not define it that way from the start, naming what is left behind “the indescribable” and deriving “indescribable” from that name? The response lies in the provenance of the three regions. Husserl’s passive synthesis comes from transcendental phenomenology, Kripke’s problem of rule-following from the philosophy of language and the foundations of mathematics, Meillassoux’s argument from contingency from speculative realism—the three advanced their analyses in mutually independent problem-domains, without reference to one another, and yet from each analysis the same structure can be extracted: “the described result is given, but the very working by which it arises retreats prior to description.” How each thinker ultimately evaluated that prior layer (as describable or not) is not uniform. What this paper relies on is not the agreement of their conclusions but the fact that three analyses of different provenance bring the same structure to light. If this paper’s claim were merely a definitional circularity, there would be no explaining why the same structure can be extracted from three inquiries of such different provenance. The independent convergence of the three is a sign that this structure is not a stipulation of this paper but a fact belonging to the activity of description itself. The three regions are not the illustration of a single conclusion but independent testimony to a single structure.

This convergence is, rather, strengthened by the fact that the mode of what is left behind is not uniform across the three. In Husserl the prior side is temporal and genetic (articulation stands after passive synthesis); in Kripke it is constitutive (the finite examples do not contain the fact that establishes which rule is followed); in Meillassoux it is modal (the description of possibility (Ω , P) does not ground that one of them becomes actual). That three prior sides of differing modality converge to the same form—“the result is given to description, but the very working that makes the result a result is not contained in the description of the result”—is an even stronger sign that this form does not derive from any particular modality such as temporality or modality, but belongs to the activity of description itself.

R-ealism describes the arisen result precisely. And it takes that description to have exhausted the world. But for a result to arise, the event of arising must precede it. And the arising itself is never contained in the description of the arisen result. Description is always after the fact—it can only record the result after the arising has finished. What the three regions show is that R-ealism’s assumption that “description closes the world” does not structurally hold. This thing left behind—the structural surplus prior to arising into meaning—is precisely the position pointed to by the imaginary dimension iD introduced in § 05. R-ealism either denies this surplus or tries to recover it as a “fact to be described eventually,” but the surplus does not vanish through denial; it persists as the very structure of description’s not closing.

4.5 The Difference from Prior Residue Accounts: The Surplus of Structure and the Surplus of Genesis

That formal, structural description fails to capture part of the real is not this paper’s invention. Prior residue accounts, represented by the qualia argument (Jackson 1982) and Russellian monism (Russell 1927), share with this paper the direction that “formal description always leaves part of the real behind.” But the status of what is left behind differs. What these point to is the very intrinsic, categorical content that fills the structure—a layer that is statically there yet does not appear in structural description. By contrast, this paper’s surplus concerns not content but genesis. What is left behind is not “what fills the structure” but “how that articulation has arisen.” Whereas the former is a static gap between structure and content, what this paper takes aim at is a genetic gap between description and arising. This difference is decisive. The intrinsic properties of Russellian monism can in principle be pointed to as “being there,” whereas this paper’s surplus, the moment it is pointed to, moves to the side of D , and the prior side retreats further (the regress argument of this section). What this paper calls iD is not a statically waiting categorical substrate but the position of the very arising that description fails to catch at each turn. A detailed genealogical comparison with prior concepts including the New-man problem and neutral monism, and a cross-cultural ordering with Eastern and Western concepts of the surplus, are left to separate work.

05 The Outside of R-ealism: The Ontological Possibility of the Extended Imaginary

The non-closure of description shown in the previous section—that description always leaves behind the surplus prior to arising into meaning—points to an “outside” that spills over the moment R-ealism’s description closes. This section sketches, in keeping with the context of this paper, the contour of Extended Imaginary Number Theory (Muranushi 2026a) as a framework that systematically situates this outside.

5.1 The Extension to C as an Ontological Model

In speaking of the incompleteness of R-ealism, this paper borrows a mathematical analogy—the extension from R to C . In mathematics, the real numbers R are situated as the real part of the complex numbers C . A complex number $z \in C$ is expressed as $z = a + bi$ ($a, b \in R, i^2 = -1$). Here a is the component complete within R , and bi is the component that

lies outside R yet is structurally joined to R . What matters is that the imaginary unit i is not R -mappable, yet is not fictitious. In complex analysis, the imaginary part has mathematical reality equal to that of the real part, and analysis on the complex plane, joining the two, is structurally richer than analysis by real numbers alone.

Extended Imaginary Number Theory (Muranushi 2026a) introduces this mathematical structure as a model for ontology. Any constituent Z of the world W is decomposed as follows.

Definition 2 · The Double Structure of Being in Extended Imaginary Number Theory

$$Z = D + iD$$

Here D is the totality of the structural aspect of the object that has, at present, arisen into meaning—taking one of the forms of observation, verbalization, conceptualization, or imagination—that is, the real dimension. iD is the structural surplus that this description always leaves behind, prior to arising into meaning—the imaginary dimension. iD is not an “unknown fact” or a “latent variable” (these belong to D the moment they can be pointed to). What iD points to is the layer prior to its being put into words.

Here we fix, as a stipulation, the term “dimension” as used in this paper. By “dimension” this paper means neither a spatial extension nor a geometric axis, nor an entity or region that bears such. A dimension is the minimal ontological form for writing, within the superposition $Z = D + iD$, an independent direction of description irreducible to any other dimension. That is, “ D and iD are dimensions” claims no more than that the two obtain simultaneously for one object Z , are irreducible to each other, and are two superposable, independent directions of description. This paper adopts “dimension” because it lets one name just three formal requirements—*independence, superposability, and minimality*—without superfluous substantive connotation. Why the surplus must be written as a dimension and nothing else is derived in § 5.2.

One point should be supplemented. What R -realism admits as real is confined to the R -mappable part of this D , while other regions of D such as quality, meaning, and value were banished outside the real as “subjective representation” (§ 2.2, § 4.1). Extended Realism first restores those articulated qualities and meanings to the real as legitimate components of D , and then situates the prior layer that the whole of D still leaves behind as iD . That is, R -realism’s closure is twofold—first it trims D down to its R -mappable part (the banishment of quality, meaning, and value), and further it denies the prior layer that the whole of D leaves behind, namely iD (the short circuit to $Z = D$). The non-closure of description seen in the previous section showed that this latter short circuit does not hold—that D cannot close by itself.

5.2 Why It Must Be iD

Here the paper must answer the weightiest question. Granting that D ’s description leaves the prior side behind (§ 04), why give that leaving-behind the dimension iD ? The claim of this section is modest—this paper does not prove the existence of iD . What it shows is the single point that, when one tries to write the surplus fixed by § 04 as a constituent of the

real, the most economical remaining option is to write it as a direction of description independent of D.

First, the surplus is not a deficiency of cognition. Were it “something not yet described but in principle describable,” it would shrink and vanish as description is extended. But as the regress argument of § 04 showed, the operation of trying to contain the arising within description produces a new prior side from where it has contained, and the surplus is reproduced at each extension of description. Moreover this iteration appears isomorphically in the three regions of different provenance—experience, meaning, event (§ 4.4)—and does not depend on the choice of description-system (§ 3.1). Since it does not stop however one changes the mode of description, the surplus belongs not to the ignorance of the one who describes but to the structure of the real itself, toward which description heads yet which it fails to catch at each turn. This is the bridge from the epistemological fact of non-closure to the ontological consequence of the reality of the surplus.

Here we dismiss the most formidable objection to this bridge. A deflationary reading—one that reads the surplus down to a limitation of representation rather than a lack on the side of the real—would say: that the surplus shrinks in no description-system is merely the self-referential limit of representation, that description cannot write its own coming-to-be from within, and not a lack on the side of the object. But this reading mistakes the location of the arising. The arising is not a state of affairs in which mere failures-to-write are added on top of an already complete world; it is the condition for there being any content articulated as D at all. If it were purely a limit on the side of representation, removing it would leave describable content remaining; but if one removes the arising, not a single content articulated into D remains. The remaining escape is to regard the arising as “itself one describable physical process,” but this is sent back to the regress argument of § 04—the moment one finishes describing that process, what is described is again one arisen result, and the prior side retreats further. The arising is reducible neither to the describable D nor to a mere shadow of representation.

Then, in what form is this surplus, which lies on the side of the real, to be written? The alternatives that might be proposed—undetermined region, horizon, layer, boundary condition, condition of generation, transcendental condition—all form one group. For all of them seek to write the surplus as “a condition or attribute subordinate to D.” But this road is wholly closed. What can be written as subordinate to D—what is fixed functionally from the values of D, or as a constraint condition of D—has, at that point, a seat inside D and becomes something that can appear in the description of D. That contradicts the very determination of the surplus as “irreducible to D.” What can be written subordinate to D is no longer a surplus.

It should be noted that a third road, distinct from both subordination and separation, might be proposed—to write the surplus neither as subordinate to D nor as set apart from D, but as a primitive co-constitutive with D. If the two mutually constitute each other, the surplus is placed neither inside nor outside D, and appears to escape the foregoing disjunction. But this road too does not escape the requirement of an independent direction of description. Co-constitution is a relation in which two terms support each other while

standing simultaneously, and for the relation to obtain, two mutually irreducible terms must in the first place stand distinguished. If one dissolves into the other, it is no longer co-constitution but reverts to subordination, and the foregoing argument takes effect again. Hence co-constitution does not eliminate the independent direction of description but rather presupposes it—it is the very relation that Extended Realism will later determine as the mutual inherence of D and iD (E1 in § 5.3), and is not a third alternative to iD .

Therefore the remaining option—and the most economical one—is to write the surplus not as subordinate to D but as a single direction of description independent of D . Moreover this direction is not set apart from D as a separate existence; it accompanied D simultaneously for the same object Z (§ § 4.1–4.3). In the superposition $Z = D + iD$, the form that satisfies this independence and co-possibility with the fewest strokes is exhausted by a single new coordinate irreducible to the existing coordinates—a single independent axis. This paper names it iD . iD is not the name of a newly discovered entity but the name given to the very state of affairs that there is no more economical way to write it than as an independent direction of description.

This configuration is isomorphic to the way that, when the real numbers R are not algebraically closed, mathematics neither demeaned the reals nor placed an undetermined vacancy, but minimally added a single axis i orthogonal to R and thereby let the system close. i lies outside R yet is not fictitious; only by joining with R does it complete the system. That addition was required by the very fact that R does not close, and is not an arbitrary appendage. The position iD occupies relative to D is no different.

The reason for binding this surplus as the single dimension iD is that experience, meaning, and event do not happen to stand side by side but have one and the same structural origin—the after-the-factness of description (§ 4.4). Were the three made into separate dimensions, the fact that they arise from one common fact would be lost from view. Yet making them a single dimension does not mean internal structurelessness. The internal articulation of iD exceeds the scope of this paper and is left to the earlier papers (Muranushi 2026b, 2026c).

Finally, regarding the ontological status of iD , we place only the minimal determination needed to avoid misunderstanding. iD is not a “non-physical substance” in dualism. Whereas in Cartesian mind-body dualism mind and body are two separated substances, in Extended Imaginary Number Theory D and iD are two dimensions of one being Z —not separated substances but structural components of one real that mutually inhere. Nor is iD a substrate posited in advance, as in neutral monism; it is a position computed back, after the fact, from the surplus that D ’s description leaves behind—that is, identified bottom-up. What this paper says positively about iD is only the relational determination of what position it occupies in complementing D ; it does not enter into its internal structure. A detailed comparison with neutral monism and dual-aspect theory, and the relation to correlationism, are left to separate work. Note also that this paper’s mathematical analogy does not claim algebraic rigor (see Muranushi 2026a, § 1.5). What it borrows is the idea of the operation of “installing a new, orthogonal, independent axis,” not the algebraic operations of complex numbers or the properties of complex analysis.

5.3 Toward Extended Realism

This paper names the ontology based on Extended Imaginary Number Theory Extended Realism, and determines it as follows.

Definition 3 · Extended Realism

Extended Realism is the position that adopts the existential structure $Z = D + iD$ of Definition 2 as its ontological foundation, distinguished from R-ealism by the following two points.

(E1) D and iD are not separated substances but mutually inhere as two dimensions of one real (difference from substance dualism).

(E2) The methodological efficacy of R-ealism is preserved as description of the domain of D , and what is rejected is solely the metaphysical short circuit $Z = D$ (relation to R-ealism).

Extended Realism does not negate R-ealism but contains it. It restores the articulated quality, meaning, and form that R-ealism discarded as “subjective representation” to the real as legitimate components of D , and then situates the “prior side of arising” that any description still leaves behind as iD on the side of the real. At the same time, it preserves the refinement of R-description that R-ealism achieved in the domain of D as a part of it - self. In this sense Extended Realism is a moderate extension of R-ealism, not a radical negation. It is an attempt to ontologically rearrange the genealogy from Galileo through Newton, Husserl, Whitehead, and Heidegger to twentieth-century physicalism, while preserving its methodological achievements. This paper does not enter into description of the internal structure of iD itself. The phenomenological structure of iD 's coming-to-obtain (Muranushi 2026b) and its existential structure of activation (Muranushi 2026c) are developed in the respective earlier papers of the series. The scope of this paper is complete at the point of pointing, from the limit of R-ealism, to its outside.

06 Conclusion: Beyond the Real

This paper has thematized one tacit metaphysical position running through natural science and ontology since the modern period—the position that equates being real with formal describability and takes as real only what can be exhaustively described, the position this paper named “R-ealism”—and analyzed the internal limit its logic runs up against. This position formed over four centuries, with Galileo’s mathematization of nature as its starting point, and attained its most explicit formulation in the reductive physicalism of the latter twentieth century and the computational worldview of the twenty-first.

But when R-ealism carries its logic to the limit, it runs up against the non-closure of description. In every region—experience, meaning, event—description can capture the arisen result, yet always leaves behind the very prior side from which it arises—the surplus prior to being articulated as meaning. This is an immanent limit arising not from external objection but from the very structure of the activity of description. Moreover this surplus is not the static intrinsic content that structural realism or Russellian monism points to, but the genetic layer from which articulation arises (§ 4.5). R-ealism tries

to recover this surplus as a “fact to be described eventually,” but the surplus persists as the very structure of description’s not closing.

This paper has presented, reconstructively, Extended Imaginary Number Theory $Z = D + iD$ (Muranushi 2026a) as a framework for systematically grasping this surplus left behind. Extended Realism, based on Extended Imaginary Number Theory, does not negate R-ealism but contains it ontologically while preserving its methodological efficacy. The cumulative success of natural science since the modern period, as the refinement of the domain of D , constitutes the best part of humanity’s intellectual achievement. What this paper has rejected is not that success but only the single step of equating it with “the complete description of the world itself”—the ontological over-extension of methodological success. The limit of R-ealism is nothing other than the point at which R-ealism, at its own extreme, necessarily points to its own outside. This paper has pointed to that point. What spreads beyond it is already developed in the Extended Imaginary Number Theory series (Muranushi 2026a, 2026b, 2026c).

* * *

In conclusion, the claim of this paper is condensed into a single sentence. R-ealism is not false, but narrow. Just as the real numbers R are merely the real part of the complex numbers C , the reality R-ealism depicts is merely the real part of a wider real.

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